I

4) $A(x) = 2x^3 - 3x^2 - 12x + 13$ $[-2, 3]$

$A'(x) = 6x^2 - 6x - 12$

$6(x^2 - x - 2)$

$6(x-2)(x+1)$

f': + 0 − 0 + over $-2, -1, 0, 2, 3$

	2	−3	−12	13
−2	2	−7	2	9
−1	2	−5	−7	20
2	2	1	−10	−7
3	2	3	−3	4

$(-2, 9)$ Rel max increase to it

$(-1, 20)$ Abs max f' test

$(2, -7)$ Abs min f' test

$(3, 4)$ Rel max increase to it

B) $B(x) = -\cos x$ $\left[\frac{-\pi}{2}, \frac{11\pi}{6}\right]$

$B'(x) = \sin x$

$B''(x) = \cos x$

f': − 0 + 0 − 0 over $-\frac{\pi}{2}, 0, \pi, 2\pi$

$\left(-\frac{\pi}{2}, 0\right)$ Rel max decrease to it

$(0, -1)$ Abs min f' test

$(\pi, 1)$ Abs max f' test

$\left(\frac{11\pi}{6}, -\frac{\sqrt{3}}{2}\right)$ Rel min decrease to it

II $x^3 + 3xy + y^3 = 1$ misapplied product rule −8pts

$3x^2 + 3y + 3x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-(x^2+y)}{x+y^2}\Big|_{(2,-1)} =$

$-\frac{(-1)}{2+1} = -1 = m$

$y + 1 = -1(x-2)$

$y = -x + 1$

C) $C(x) = (a+x)\sqrt{a^2-x^2}$ (u, v) no product rule −15

$C'(x) = \sqrt{a^2-x^2}' + \frac{(a+x)(-x)}{\sqrt{a^2-x^2}}$

implied and pts for $\sqrt{\ }$ exist $\pm a$

$= \frac{a^2 - x^2 - x(a+x)}{\sqrt{a^2-x^2}}$

$(-a, 0)$ Abs min from inc

$= \frac{(a+x)(a-x) - x(a+x)}{\sqrt{a^2-x^2}}$

$\left(\frac{a}{2}, \frac{3a^2\sqrt{3}}{4}\right)$ Abs max f' test

$= \frac{(a+x)(a-2x)}{\sqrt{a^2-x^2}}$

(sing anax)

$\frac{3a}{2}\sqrt{a^2 - \frac{c^2}{4}}$

$\frac{3a}{2}\left(\frac{a}{2}\sqrt{3}\right)$

$\frac{3a^2\sqrt{3}}{4}$

$x \in \{-a, \frac{a}{2}\}$

f': − 0 + 0 − over $-a, 0, \frac{a}{2}, a$

$(a, 0)$ Abs min decrease to it

III A) $\dfrac{(2x+1)^3(4-x^2)^4}{(8x^3-3)^2}$ q / r

$$\frac{(8x^3-3)^2\left[(4-x^2)^4\,3(2x+1)^2(2)+(2x+1)^3\,4(4-x^2)^3(-2x)\right]-(2x+1)^3(4-x^2)^4\,2(8x^3-3)(24x^2)}{(8x^3-3)^4}$$

B) $(3x-2)^{\frac{2}{3}}(4-x)^{\frac{1}{3}}\sqrt{3x^3-1}$ u / v

$$\sqrt{3x^3-1}\left[(4-x)^{\frac{1}{3}}\tfrac{2}{3}(3x-2)^{-\frac{1}{3}}(3)+(3x-2)^{\frac{2}{3}}\tfrac{1}{3}(4-x)^{-\frac{2}{3}}(-1)\right]+(3x-2)^{\frac{2}{3}}(4-x)^{\frac{1}{3}}\tfrac{1}{2}(3x^3-1)^{-\frac{1}{2}}9x^2$$

Extra.

A) $A=2x^3-3x^2-12x+13$

$A'(x)=6x^2-6x-12$

$A''(x)=12x-6$

$6(2x-1)$ $\quad x=\frac{1}{2}$

$2\cdot\frac{1}{8}-3\cdot\frac{1}{4}-6+13$

$13-6\frac{1}{2}$

$6\frac{1}{2}$

$\frac{13}{2}$

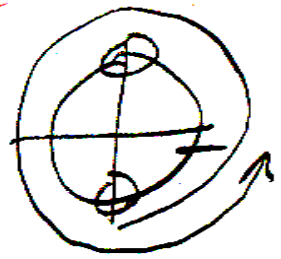
A'' : − 0 + (0, $\frac{1}{2}$)

PI $(\frac{1}{2},\frac{13}{2})$

B) $B(x)=-\cos x$

$B'(x)=\sin x$

$B''(x)=\cos x$



$-\frac{\pi}{2}$ is END pt. ∴ not PI

$(\frac{\pi}{2},0)$, $(\frac{3\pi}{2},0)$ } PI

B'' : + 0 − 0 + (0, $\frac{\pi}{2}$), ($\frac{3\pi}{2}$, $\frac{11\pi}{6}$)

2000

100	1	0
90	8	5
80	4	5
70	2	0
60	4	2
59	1	1

101

1658 / 20 | 1088 / 13 | 2746 / 33 | 55

82.9 | 83.69 | 83.2

A really easy exam
those who were keen on
Board Exam & those who
didn't it really work
David Thomas.

91

	2	5	T
100	0	0	0
90	8	8	16
80	11	8	19
70	2	3	5
60	0	1	1
59	1	0	1

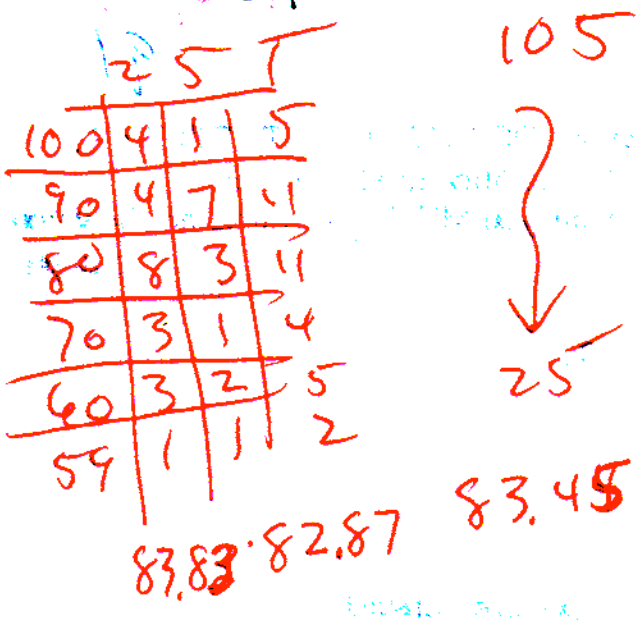
99

1865 / 22 | 1837 / 21 | 3698 / 43 | 35

84.77 | 87.29 | 86.0

Excellent results.
requiring "factor" on
section II (mainly)
and regard was good
Practice and focused
students.

89



I should have required

lines showing
change of comment
before giving credit on
the test for credit.

Honors Analysis Name AWL ch 4m

Copy the original problem. Per _____ Date 4/14/2000
Convince *me* that **you** understand the concept.

CHAPTER #4 Mechanical

I Find and identify the *coordinates* of all extrema for each of the following. Do NOT draw the graph of the function. Each extrema must be labeled as "relative" or "absolute" in addition to Maximum or Minimum. Remember that end points yield extrema **and** that you *must justify* (explain how you know) they are what you say they are. (20 pts ea)

a) $A(x) = 2x^3 - 3x^2 - 12x + 13 \qquad x \in [-2, 3]$

b) $B(x) = -\cos x \qquad x \in \left[\frac{-\pi}{2}, \frac{11\pi}{6}\right]$

c) $C(x) = (a+x)\sqrt{a^2 - x^2}$ "a" is a positive constant
(hint: What are the domain restrictions on the function?)

II Given: $x^3 + 3xy + y^3 = 1$ (20 pts)

Find the equation of the line tangent to the curve at $(2, -1)$

III Given $f(x)$, determine $f'(x)$. Do not simplify your answers. (for example, leave negative exponents exponents.) Basically, I am looking for evidence that you have used the proper formula at the proper time. (10 pts ea)

a) $\dfrac{(2x+1)^3(4-x^2)^4}{(8x^3-3)^2}$

b) $\left((3x-2)^{\frac{2}{3}}(4-x)^{\frac{1}{3}}\right)\sqrt{3x^3-1}$

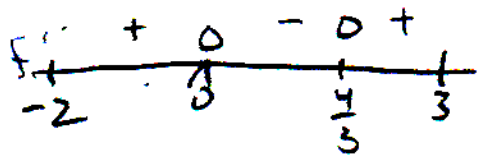
Extra Credit **5 pts**

Determine the *coordinates* of all points of inflection for the functions given in Section I a and I b. Show appropriate number lines as justification.

I a) $f(x) = 2x^3 - 4x^2$ $[-2, 3]$

$f'(x) = 6x^2 - 8x$

$2x(3x-4)$



$f(-2) = -16 - 16 = -32$

$f(0) = 0$

$f(\frac{4}{3}) = 2 \cdot \frac{64}{27} - 4 \cdot \frac{16}{9}$ $\quad \frac{64}{3}$

$\frac{128}{27} - \frac{192}{27} = -\frac{64}{27}$

$f(3) = 2 \cdot 27 - 4 \cdot 9$

$54 - 36 = 18$

$(-2, -32)$ min end pt ABS
inc fun it
$(0, 0)$ max f' Rel
$(\frac{4}{3}, -\frac{64}{27})$ min f' Rel
$(3, 18)$ max end pt ABS
inc to it.

b) $f(x) = 2\cos x(\sin x - 1)$ $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

$f'(x) = -2\sin x(\sin x - 1) + 2\cos^2 x$

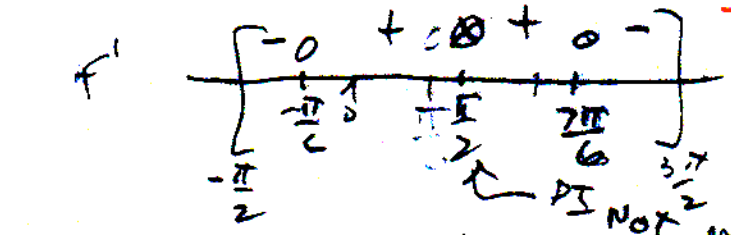
$= -2\sin^2 x + 2\sin x + 2(1 - \sin^2 x)$

$= -4\sin^2 x + 2\sin x + 2$

$-2(2\sin^2 x - \sin x - 1)$ Must The factored -1 PTS

$-2(2\sin x + 1)(\sin x - 1)$

$\sin x = -\frac{1}{2}$ $\quad \sin x = 1$ non part pt! -5 PTS



$F(0) = -2(1)(-1) > 0$ max nor min.

$f(-\frac{\pi}{2}) = 2 \cdot 0(\ ) = 0$

$f(-\frac{\pi}{6}) = 2 \cdot \frac{+\sqrt{3}}{2}(\frac{-1}{2} - 1) = -\frac{3\sqrt{3}}{2}$

$f(\frac{7\pi}{6}) = 2(-\frac{\sqrt{3}}{2})(\frac{-1}{2} - 1) = \frac{3\sqrt{3}}{2}$ ABS

$f(\frac{3\pi}{2}) = 0$

$(-\frac{\pi}{2}, 0)$ Rel end pt inc fun it

$(-\frac{\pi}{6}, -\frac{3\sqrt{3}}{2})$ min f' test

$(\frac{7\pi}{6}, \frac{3\sqrt{3}}{2})$ max f' test ABS

$(\frac{3\pi}{2}, 0)$ min end pt dec Rel to it

II $x^4 + 2x^2y + y^2 = 4$

$4x^3 + y \cdot 4x + 2x^2\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{(4x^3 + 4xy)}{2x^2 + 2y}$

$= -\frac{2x(x^2 + y)}{x^2 + y}$

$\frac{dy}{dx} = -2x$

$\left.\frac{dy}{dx}\right|_{x=1} = -2 = m$

$x = 1$

$1 + 2y + y^2 = 4$

$y^2 + 2y - 3 = 0$

$(y + 3)(y - 1) = 0$

$y \in \{1, -3\}$

$(1, 1)$ $m = -2$

$y - 1 = -2(x - 1)$

$y = -2x + 3$

$(1, -3)$ $m = -2$

$y + 3 = -2(x - 1)$

$y = -2x - 1$

III

a) $(fg)'\big|_{x=2} = gf' + fg'$

$3 \cdot 12 + 3(-2)$

$36 - 6$

30

no product rule

b) $\left(\frac{f}{f+g}\right)'\Big|_{x=3}$ $\quad \frac{(f+g)f' - f(f'+g')}{(f+g)^2}$

$\frac{(5+2)12 - 5(12-3)}{(5+2)^2}$

no quotient rule

$\frac{84-45}{49} = \frac{39}{49}$

c) $(f(g))'\big|_{x=3} = f'g \cdot g'(x)$

$f'(2) \cdot (4'3)$

$12(-3)$

-36

IV $f(x) = \frac{2(9x-2)}{135}\sqrt{(3x+1)^3}$ (u, v)

$(3x+1)^{\frac{3}{2}}\frac{18}{135} + \frac{x(9x-2)}{135} \cdot \frac{3(3x+1)^{\frac{1}{2}}}{x} \cdot 3$

$\frac{(3x+1)^{\frac{1}{2}}}{135}\left[18(3x+1) + 9(9x-2)\right]$

$\frac{(3x+1)^{\frac{1}{2}}}{135}\left[54x + 18 + 81x - 18\right]$

$135x$

$x(3x+1)^{\frac{1}{2}}$

$x\sqrt{3x+1}$

extra.

from II

$\frac{dy}{dx} = -2x \overset{\text{set}}{=} 0$

$x = 0$

$x^4 + 2x^2y + y^2 = 4$

$y^2 = 4$

$y = \pm 2$

coord where tangent is horizontal are $(0,2)$ and $(0,-2)$

200

	4	5	T
100	0	0	0
90	1	4	5
80	3	6	9
70	6	2	8
60	5	5	10
54	2	4	6

97

40

$\frac{1236}{17}$ $\frac{1575}{21}$ $\frac{2862}{38}$

72.7 75.0 74.0

an excellent exam.
I think too many are trying to memorize how to do the problem rather than learning the 3 or 4 situations and how to deal with any combination of them.
a 2nd point is wrong didn't or couldn't solve the trig equation I'll a la freeform rings. This serves as an excellent wake up call.
No changes necessary.

Honors Analysis

Name ANL Ch4M

Per _____ Date 2007

Copy original problem.

Convince *me* that **you** understand the concept!

No Calculators.

Chapter 4 – Mechanical Exam

I Find and identify (including "relative" or "absolute") the coordinates of all extrema for each of the following. Remember to consider the domain limits. Do **not** draw the graph of the function. All conclusions must be properly justified. Be *very* sure the reader understands your logic!

(25 pts ea)

a) $f(x)=2x^3-4x^2 \quad x \in [-2,3]$

b) $f(x)=2\cos x(\sin x-1) \quad x \in [-\frac{\pi}{2},\frac{3\pi}{2}]$

II Given: $x^4+2x^2y+y^2=4$. Find the equations of the tangent lines at $x=1$ (20 pts)

III Given the following table with function values:

$x\rightarrow$	2	3
$f(x)$	3	5
$f'(x)$	12	12
$g(x)$	3	2
$g'(x)$	−2	−3

$(fg)'\big|_{x=2}$ translates to: "the derivative of the product of f and g evaluated at x = 2"

Evaluate: (20 pts tot)

a) $(fg)'\big|_{x=2}$

b) $\left(\frac{f}{f+g}\right)'\Big|_{x=3}$

c) $(f(g))'\big|_{x=3}$

IV Given: $f(x)=\frac{2(9x-2)}{135}\sqrt{(3x+1)^3}$. Find $f'(x)$ (simplify completely). (10 pts)

Extra Credit ---------- **5 pts** ----------

Reconsider the equation given in II above. Find any and all coordinates where the tangent line would be horizontal.

I a) $x^3+y^3=3xy$ — NO Product rule −10

$3x^2+3y^2\frac{dy}{dx}=3y+3x\frac{dy}{dx}$

$(3y^2-3x)\frac{dy}{dx}=3y-3x^2$

$\frac{dy}{dx}=\frac{y-x^2}{y^2-x}$

$(\frac{2}{3},\frac{4}{3})$

$y-y_1=m(x-x_1)$

$y-\frac{4}{3}=\frac{4}{5}(x-\frac{2}{3})$

$y-\frac{4}{3}=\frac{4}{5}x-\frac{8}{15}$

$15y-20=12x-8$

$12x-15y+12=0$

$4x-5y+4=0$ ★

b) $4x-5y+4=0$ — if set $y=y$ set must be used 1PT

$(x+y+1=0)5$

$5x+5y+5=0$

$9x+9=0$

$x=-1$

$y=0$

the coordinate of intersection is $(-1,0)$

$\frac{\frac{1}{3}-\frac{4}{9}}{\frac{16}{9}-\frac{2}{8}}=\frac{12-4}{16-6}$ $\quad \frac{8}{10}=\frac{4}{5}$

II $f(x)=\frac{x}{1-x}$ a) D_f $x\neq 1$

$1-x\overset{\text{set}}{=}0$

$x=1$

R_f $f(x)\neq -1$ 3pts

$x=\frac{y}{1-y}$

$x-xy=y$

$y(1+x)=x$

$y=\frac{x}{1+x}$

$1+x\overset{\text{set}}{=}0$

$x=-1$

b) $f'(x)=\frac{1-x-x(-1)}{(1-x)^2}$

$=\frac{1-x+x}{(1-x)^2}$

$=\frac{1}{(1-x)^2}$

$f'(3)=\frac{1}{4}$

$f(3)=\frac{3}{1-3}=-\frac{3}{2}$

$y+\frac{3}{2}=\frac{1}{4}(x-3)$

$y+\frac{3}{2}=\frac{1}{4}x-\frac{3}{4}$

$y=0$

$\frac{9}{4}=\frac{1}{4}x$

$x=9$.

x intercept

9. ★

c) $\perp$ 1PT

$y=\frac{1}{(1-x)^2}$ — don't set here −4pts

$(1-x)^2=\frac{1}{4}$

$1-x=\pm\frac{1}{2}$

$1\pm\frac{1}{2}=x$

$x=\{\frac{3}{2},\frac{1}{2}\}$

$|m|=2$

$x+4y-2=0$

$y=-\frac{1}{4}x+\frac{1}{2}$

$\perp m=4$

$\frac{\frac{3}{2}}{1-\frac{3}{2}}=\frac{\frac{3}{2}}{-\frac{1}{2}}=-3$

$(\frac{3}{2},-3)$ $m=4$

$y+3=4(x-\frac{3}{2})$

$4x-6$

★ $y=4x-9$

$\frac{\frac{3}{2}}{1-\frac{1}{2}}=1$

$(\frac{1}{2},1)$ $m=4$

$y-1=4(x-\frac{1}{2})$

$=4x-2$

★ $y=4x-1$

III $f(x)=\sin^2 x-\sin x$ $\quad 0\le x\le \frac{3\pi}{2}$

$f'(x)=2\sin x\cos x-\cos x$

$\cos x(2\sin x-1)\overset{\text{set}}{=}0$

Wrong der −15

Right but unable to factor −10

$f'(\frac{\pi}{8})>0$ ∴

$\frac{1}{4}-\frac{1}{2}$

	0		+		+		−		0
f'	0	$\frac{\pi}{6}$		$\frac{\pi}{2}$		$\frac{5\pi}{6}$		π	$\frac{3\pi}{2}$

$-(4\sin^2 x-\sin x-2)$

(0,0) endpt def'n'd max Local

$(\frac{\pi}{6},-\frac{1}{4})$ min f'test abs

$(\frac{\pi}{2},0)$ max f'test Local

$(\frac{5\pi}{6},-\frac{1}{4})$ min f'test. labs

$(\frac{3\pi}{2},2)$ max end pt abs inc to it,

$(\frac{3\pi}{2},2)$

graph not required.

U $f(x) = \dfrac{3x^2\sqrt{2x^2-3x+1}}{(4x-3\sqrt{x})^3\sqrt{3x^2+2x+1}}$ $\begin{matrix}u\\ v\end{matrix}$

$$\frac{(4x-3\sqrt{x})^3\sqrt{3x^2+2x+1}\left[\sqrt{2x^2-3x+1}\cdot 6x + 3x^2\cdot\frac{1}{2}(2x^2-3x+1)^{-\frac{1}{2}}(4x-3)\right] - (3x^2\sqrt{2x^2-3x+1})\left[\sqrt{3x^2+2x+1}\,3(4x-3\sqrt{x})^2\left(4-\frac{3}{2}x^{-\frac{1}{2}}\right)+(4x-3\sqrt{x})^3\frac{1}{2}(3x^2+2x+1)^{-\frac{1}{2}}(6x+2)\right]}{\left((4x-3\sqrt{x})^3\sqrt{3x^2+2x+1}\right)^2}$$

EXTRA.

Line from part I$_A$ $\quad 4x-5y+4=0$

formula $\dfrac{|c|}{\sqrt{A^2+B^2}} \rightarrow \dfrac{4}{\sqrt{16+25}}$

$\dfrac{4}{\sqrt{41}}$

$\dfrac{4\sqrt{41}}{41}$

4/19/02

	3	5	T
100	0	0	0
90	3	2	5
80	6	6	12
70	6	6	12
60	0	8	8
59	3	4	7

99

78.7 73.3 75.6

32

Excellent Exam.
This was largely the 1999 Exam. most kids'll Practice it.

This exam took a very long time to grade!!
(about 6-7 hrs.!)

4/23/99

	4	5	I	
100	0	0	0	94
90	0	1	1	
80	3	3	6	
70	4	5	9	
60	3	3	6	
59	4	5	9	42

954	1153	2107
14	17	31
68.1	67.8	68.0

An excellent exam.
The sad part is that this would
wasted on such poorly
prepared students.
The STAR testing took a lot of
a toll. The SBCP day was the
Big hit, however. We didn't "do
hard 4/5 exam" because of it.

Student Body
Carnival for
the students'
effort on the
STAR test
Sun.

Honors Analysis Name AUL Ch 4 Mech

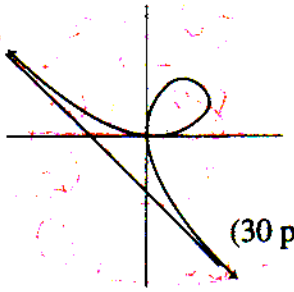
Copy original problem. Per _____ Date 2007

Convince *me* that you understand the concept!

No calculators.

Chapter 4 Mechanical Exam

I The curve on the right is called the *folium of Decartes* and has the equation: $x^3 + y^3 = 3xy$. The curve was originally proposed by Rene Decartes as a challenge to Pierre Fermat to find its tangent line.
The curve has as a tilted asymptote $y = -x - 1$.



(30 pts)

a) Find that tangent line at $(\frac{2}{3}, \frac{1}{3})$. Write the equation of the line in *standard* form, ie. without fractions and equal to zero.

b) Find the coordinate where the tangent line in part (a) intersects with the asymptote.

II Given $f(x) = \dfrac{x}{1-x}$. (hint: Be sure your responses are properly justified.) (tot 30 pts)

a) Find the domain and range of $f(x)$.

b) Determine the x-intercept of the tangent line to $f(x)$ at $x = 3$.

c) Find the equation(s) of all tangent lines to $f(x)$ which are perpendicular to: $x + 4y - 2 = 0$

III Identify (and justify) the coordinates of ***all*** extreme points for the given function. Use the phrase "absolute" or "local" as appropriate. (20pts)

$$f(x) = \sin^2 x - \sin x \quad \text{for } 0 \le x \le \frac{3\pi}{2}.$$

IV Given $f(x)$. Determine $f'(x)$. Do not simplify your answers. (for example, leave negative exponents as negative exponents, do not combine constants, etc.) Basically, I am looking for evidence that you have used the proper formula in the proper order at the proper time. (20 ps)

$$f(x) = \frac{3x^2\sqrt{2x^2 - 3x + 1}}{(4x - 3\sqrt{x})^3\sqrt{3x^2 + 2x + 1}}$$

Extra Credit ---------- 5 pts ----------

Reconsider the folium of Decartes in section I. What is the shortest distance from the origin to the line tangent to the folium at $(\frac{2}{3}, \frac{1}{3})$?

I

$f(x) = \frac{3x(2x-1)^4}{5x+2}$ $x \in [-2, 2]$

$f'(x) = \frac{(5x+2)\left[(2x-1)^4 3 + 3x(2)\right] - 3x(2x-1)5}{(5x+2)^2}$

$= \frac{(5x+2)(6x-3+6x) - 15x(2x-1)}{}$

$= 3(5x+2)(4x-1) - 15x(2x-1)$

$= 3(20x^2+8x-2-10x^2+5x)$

$= 6[5x^2+4x-1]$

$= \frac{6(5x-1)(x+1)}{(5x+2)^2}$ Not factored -10

$f'(x)$: + 0 − ∄ − 0 + E

$-2, -1, -\frac{2}{5}, \frac{1}{5}, 2$

$f(-2) = \frac{-6(-5)}{-8} = -\frac{15}{4}$

$f(-1) = \frac{-3(-3)}{-3} = -3$

$f(\frac{1}{5}) = \frac{\frac{3}{5}(-\frac{3}{5})}{\frac{11}{5}} = -\frac{3}{25}$

$f(2) = \frac{6(3)}{12} = \frac{3}{2}$

$(-2, -\frac{15}{4})$ min ENDPT INC fun't.

$(-1, -3)$ max f 'test'

$(\frac{1}{5}, -\frac{3}{25})$ min f 'test'

$(2, \frac{3}{2})$ max ENDPT INC to CPT.

II

a) $x = 2$ $3x^2 - 2xy - y^2 = -9$

$3 \cdot 4 - 4y - y^2 = -9$

$y^2 + 4y - 21 = 0$

$(y+7)(y-3) = 0$

$y \in \{-7, 3\}$

No product rule -12

$(2, 3)$

$(2, -7)$

b) $3x^2 - 2xy - y^2 = -9$ No zero -5

$6x - 2y - 2x\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$ ↓ (if y were $-\frac{3}{5}x$, $\frac{dy}{dx} = 0$)

$\frac{y-3x}{-(x+y)} = \frac{dy}{dx} = \frac{3x-y}{x+y}$

$(2, 3)$: $\frac{6-3}{2+3} = \frac{3}{5} = m$ $y - 3 = \frac{3}{5}(x-2)$

$y = \frac{3}{5}x - \frac{6}{5} + \frac{15}{5}$

$y = \frac{3}{5}x + \frac{9}{5}$

$(2, -7)$: $\frac{-7-7}{2-7} = -\frac{13}{5} = m$ $y + 7 = -\frac{13}{5}(x-2)$

$y = -\frac{13}{5}x + \frac{26}{5} - \frac{35}{5}$

$y = -\frac{13}{5}x - \frac{9}{5}$

c) $\frac{3}{5}x + \frac{9}{5} \overset{\text{set}}{=} -\frac{13}{5}x - \frac{9}{5}$

$3x + 9 = -13x - 9$

$16x = -18$

$x = -\frac{9}{8}$

$\frac{3}{5}(-\frac{9}{8}) + \frac{9}{5}$

$-\frac{27}{40} + \frac{72}{40}$

$\frac{45}{40}$

$\frac{9}{8}$

intersection $(-\frac{9}{8}, \frac{9}{8})$

III $f(x) = \sin 2x - 2\cos x \quad -\pi \le x \le \pi$

$f'(x) = 2\cos 2x + 2\sin x$

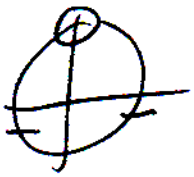
$2(\cos^2 x - \sin^2 x + \sin x)$

$2(1 - 2\sin^2 x + \sin x)$

$-2(2\sin^2 x - \sin x - 1)$

$-2(2\sin x + 1)(\sin x - 1)$

$-\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{2}$



f': $+ \; 0 \; - \; 0 \; + \; \oslash \; +$ at $-\pi, -\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{\pi}{2}, \pi$

f'': $+ \; 0 \; - \; 0 \; +$ at $-\pi, -\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi$

$f'' = -4\sin 2x + 2\cos x$

$-4(2\sin x \cos x + 2\cos x)$

$-8\cos x(\sin x + 1)$

$-\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$

$\frac{3\pi}{2}$: nonparticip

$-\frac{\pi}{2}$: "part." bc does participate

$x =$

$-\pi$ min endpt inc fcn if.

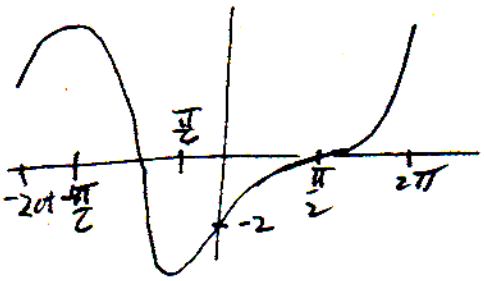
$-\frac{5\pi}{6}$ max f' chg

~~$-\frac{\pi}{2}$ pI up → dn~~

$-\frac{\pi}{6}$ min f' chg

~~$\frac{\pi}{2}$ LPI dn → up~~

π max endpt inc to it



IV $f(x) = \left(\frac{2\sin x \cos x}{\cos^2 x - \sin^2 x}\right)^3$ (u, v)

$f'(x) = 3\left(\frac{2\sin x\cos x}{\cos^2 x - \sin^2 x}\right)^2 \left[\frac{(\cos^2 x - \sin^2 x)(\cos x \cdot 2\cos x + 2\sin x(-\sin x)) - (2\sin x\cos x)(2\cos x(-\sin x) - 2\sin x\cos x)}{(\cos^2 x - \sin^2 x)^2}\right]$

$\tan\frac{\pi}{3}$ $\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$

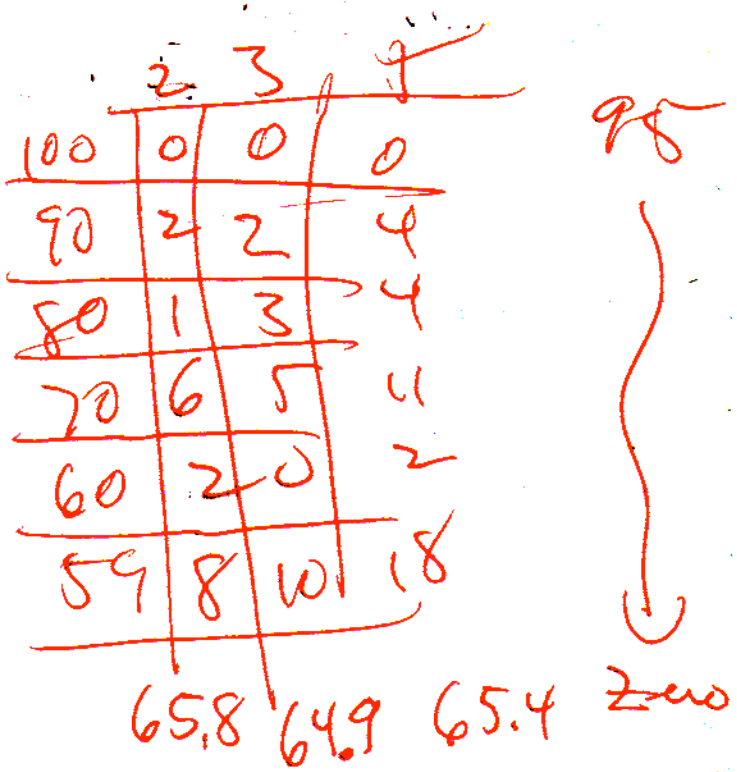
Ex $f(x) = \left(\frac{\sin 2x}{\cos 2x}\right)^3$

$= \tan^3 2x$

$f'(x) = 3\tan^2 2x \sec^2 2x \cdot 2$

$f'\left(\frac{\pi}{6}\right) = 3 \cdot 3 \cdot 4 \cdot 2$

$= 9 \cdot 8 = 72$



Essthe break then start test.

Argh!!

these kids need a foot in their butt Daily or they get super sloppy.

I should have made them do Quizzes on class days because they didn't take advantage of the very limited time. The very limited time.

Honors Analysis

Copy original problem.

Convince *me* that **you** understand the concept!

No calculators.

Name Ch 4 m

Per _____ Date 2003

Chapter 4 Mechanical Exam

I Given $f(x)=\dfrac{3x(2x-1)}{5x+2}$. Determine and identify the (x,y) coordinates of all extrema between $x=-2$ and $x=2$ inclusive. Justify, of course. (25 pts)

II Given the curve: $3x^2-2xy-y^2=-9$ and the line $x=2$. (30 pts tot)

a) Find the coordinates of intersection of the line and the curve.

b) Find the equation of each of the tangent lines to the curve at those points.

c) Find the coordinates of the intersection of the tangent lines.

III Given: $f(x)=\sin 2x-2\cos x$ and $-\pi\le x\le\pi$. Find, identify and justify the abscissa values of all maximum, mimimum and PI. Sketch the graph illustrating the significant points. (25 pts)

IV Given $f(x)=\left(\dfrac{2\sin x\cos x}{\cos^2 x-\sin^2 x}\right)^3$. (tot 20 pts)

Determine $f'(x)$. Do not simplify your answers. (For example, leave negative exponents as negative exponents, do not combine constants, etc.) I am looking for evidence that you have used the proper derivative formula at the proper time.

Extra Credit ---------- 5 pts ----------

Using the function in section IV above, Find $f'\left(\frac{\pi}{6}\right)$..

Honors Analysis Chapter 4

"Mechanical"

2003

				Exam Chapter 3 3 / 28
Important Lecture # 1 Part I A, B, C **Do even numbered problems ONLY** 3 / 31	**Industrial Strength Derivatives # 2** Part I A, B, C **Do odd numbered problems ONLY** 4 / 1	**Important Lecture** # 3, 4 ISD section II section II Part A & B 4 / 2	Minimum Day # 5 Unchained Melody 4 / 3	Minimum Day **Important Lecture # 6 ISD section III** 4 / 4
# 7 ISD IV V problems a - i 4/ 7	**# 8 ISD V problems j - r** 4/ 8	**# 9 ISD VI, VII a,b** 4/ 9	**# 10, 11 VII c, d, e, f** 4/ 10	Begin Spring Break work Due 4/21 #12, 13, 14 "Newton Searches for Easter Eggs" 4/ 11
Spring Break	**also during break, *BEGIN***		**# 15, 16, 17, 18, 19 "Heston's Derivative Marathon"**	
¿Did somebody parlez el Quizzo?	une autra la quizzette????		Put Last Year's Exam on Chalkboards	**Exam Chapter 4 "Mechanical" 2003**
4/ 21	4/ 22	4/ 23	4/ 24	4/ 25

"Heston's Derivative Marathon" is due end of period 4/23

Now you can take the derivative of almost anything. In the next unit we shall find out what we can do with these derivatives. Does it sound like we're heading toward devine word problems? It should. We love Analysis!!

$2x^3(x-2)$

I A) $f(x) = 2x^4 - 4x^3$ $[-1, 2]$

$f'(x) = 8x^3 - 12x^2$

$4x^2(2x-3)$

f': $-$ ∄ $-$ 0 $+$ $\frac{3}{2}$ $+$

$f(-1) = -2(-3) = 6$

$f(\frac{3}{2}) = 2 \cdot \frac{27}{8} \cdot (-\frac{1}{2}) = -\frac{27}{8}$

$f(2) = 16 \cdot 0 = 0$

$(-1, 6)$ Abs max def'mt

$(\frac{3}{2}, -\frac{27}{8})$ Abs min f' DNE

$(2, 0)$ Rel max INC to it

B) $f(x) = \sin x + \cos x$ $\quad -\frac{\pi}{3}, \frac{5\pi}{3}$

$f'(x) = \cos x - \sin x \overset{\text{Set}}{=} 0$

$\sin x = \cos x$

$x \in \{\frac{\pi}{4}, \frac{5\pi}{4}\}$

f': $+$ 0 $-$ 0 $+$ $\quad -\frac{\pi}{3}$ 0 $\frac{\pi}{4}$ $\frac{5\pi}{4}$ $\frac{5\pi}{3}$

$f(-\frac{\pi}{3}) = -\frac{\sqrt{3}}{2} + \frac{1}{2}$ $\quad (-\frac{\pi}{3}, \frac{1-\sqrt{3}}{2})$ Rel min INC fn it

$f(\frac{\pi}{4}) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}$ $\quad (\frac{\pi}{4}, \sqrt{2})$ Abs max f' DNE

$f(\frac{5\pi}{4}) = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}$ $\quad (\frac{5\pi}{4}, 0)$ Abs min f' DNE

$f(\frac{5\pi}{3}) = -\frac{\sqrt{3}}{2} + \frac{1}{2}$ $\quad (\frac{5\pi}{3}, \frac{1-\sqrt{3}}{2})$ max Rel INC to it

Question is $-\sqrt{2} < \frac{1-\sqrt{3}}{2}$?

Both negative, if squared, the most neg will be larger.

$2, \frac{2(2-\sqrt{3})}{2}$. clearly, $2-\sqrt{3}$ is smaller than 2.

II $y^2 + 3xy - x^3 = -1$ $\quad u \quad v$

$y^2 + 6y - 8 = -1$

$y^2 + 6y - 7 = 0$

$(y+7)(y+1) = 0$

$y \in \{-7, +1\}$

$2y\frac{dy}{dx} + 3y + 3x\frac{dy}{dx} - 3x^2 = 0$

$\frac{dy}{dx} = \frac{3x^2 - 3y}{3x + 2y}$

$\left.\frac{dy}{dx}\right|_{\substack{x=2 \\ y=1}} = \frac{12+3}{6+2} = \frac{9}{8}$

$y - 1 = \frac{9}{8}(x-2)$

$y - 1 = \frac{9}{8}x - \frac{9}{4}$

$y = \frac{9}{8}x - \frac{5}{4}$

$\left.\frac{dy}{dx}\right|_{\substack{x=2 \\ y=-7}} = \frac{12+21}{6-14} = -\frac{33}{8}$

$y + 7 = -\frac{33}{8}(x-2)$

$= -\frac{33}{8}x + \frac{33}{4}$

$y = -\frac{33}{8}x + \frac{5}{4}$

III

a) $(\overset{u}{(2-3x)^{\frac{2}{3}}}\overset{v}{(4x+1)^{\frac{3}{2}}})\overset{\cdot}{\sqrt{5-2x^3}}$

$\sqrt{5-2x^3}\left[(4x+1)^{\frac{3}{2}}\cdot\frac{2}{3}(2-3x)^{-\frac{1}{3}}(-3)+(2-3x)^{\frac{2}{3}}\cdot\frac{3}{2}(4x+1)^{\frac{1}{2}}4\right]+(2-3x)^{\frac{2}{3}}(4x+1)^{\frac{3}{2}}\frac{1}{2}(5-2x^3)^{-\frac{1}{2}}(-6x^2)$

b) $\dfrac{(\overset{u}{8x^3-3x})^{2}(\overset{v}{2x+1})^{3}}{(4-x^2)^4}$ $\quad u\quad v$

$$\frac{(4-x^2)^4\left[(2x+1)^3 2(8x^3-3x)(24x^2-3)+(8x^3-3x)^2\,3(2x+1)^2(2)\right]-(8x^3-3x)^2(2x+1)^3 4(4-x^2)^3(-2x)}{\left[(4-x^2)^4\right]^2}$$

IV $f(x)=x^2-10 \quad x_1=3$

$f'(x)=2x$

Newton's method: $x_{N+1}=x_N-\dfrac{f(x_N)}{f'(x_N)}$

$x_2=3-\dfrac{9-10}{6}$

$=3+\frac{1}{6}$

$=\frac{19}{6}$

Extra $\quad f(x)=\cos x$

$f'=\lim_{h\to 0}\dfrac{\cos(x+h)-\cos x}{h}$

$\lim_{h\to 0}\dfrac{\cos x\cos h-\sin x\sin h-\cos x}{h}$

$\lim_{h\to 0}\dfrac{-\cos x(1-\cos h)}{h}-\dfrac{\sin h}{h}\cdot\sin x$

$\cos x\cdot\dfrac{1-\cos h}{h}\to 0 \qquad \dfrac{\sin h}{h}\to 1$

$f'(x)=-\sin x.$

must use limit

98

	2	5	7
100	3	2	5
90	5	5	10
80	7	5	12
70	1	4	5
60	2	1	3
59	3	21	5

105

34

$\frac{1726}{21}$ $\frac{1576}{19}$

82.2 82.9 82.6

Honors Analysis

Name ANL Ch 4 M

Copy original problem.

Per _____ Date 93

Convince *me* that **you** understand the concept!

Chapter 4 – Mechanical Exam

I Find and identify (including "relative" or "absolute") the coordinates of all extrema for each of the following. Remember to consider the domain limits. **Do not draw the graph of the function.** All conclusions must be properly justified. Be *very* sure the reader understands your logic! (25 pts ea)

a) $f(x) = 2x^4 - 4x^3 \quad x \in [-1,2]$

b) $f(x) = \sin x + \cos x \quad x \in [-\frac{\pi}{3}, \frac{5\pi}{3}]$

II Given: $y^2 + 3xy - x^3 = -1$. Find an equation *for each* of the tangent lines when $x = 2$. (20 pts)

III Given $f(x)$. Determine $f'(x)$. **Do not simplify** your answers. (For example, leave negative exponents as negative exponents.) Basically I am looking for evidence that you have used the proper formula at the proper time. (10 pts ea)

a) $(2-3x)^{\frac{2}{3}}(4x+1)^{\frac{3}{2}}\sqrt{5-2x^3}$

b) $\dfrac{\left(8x^3-3x\right)^2(2x+1)^3}{\left(4-x^2\right)^4}$

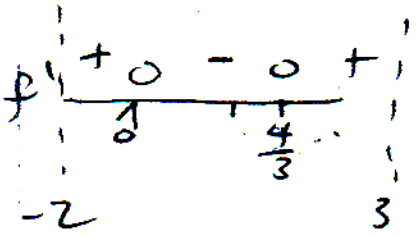
IV Given: $f(x) = x^2 - 10$ and $x_1 = 3$. Find x_2 as described by Newton's method (for finding roots). (10 pts)

Extra Credit ---------- 5 pts ----------

Using the definition of the derivative which yields a function, prove: $(\cos x)' = -\sin x$

I 4) $2x^2(x-2)$

4) $f(x) = 2x^3 - 4x^2$



$f'(x) = 6x^2 - 8x$

$2x(3x-4) \overset{\text{Set}}{=} 0$

$f(-2) = 2(4)(-4)$ $(-2, -32)$ min inc fn it Abs

$f(0) = 0$ $(0, 0)$ max f' test. Rel

$f(\frac{4}{3}) = 2 \cdot \frac{16}{9}(-\frac{2}{3})$ $(\frac{4}{3}, -\frac{64}{27})$ min f' test. Rel.

$-\frac{64}{27}$

$f(3) = 2 \cdot 9 (1) = 18$ $(3, 18)$ max inc to it Abs

b) $f(x) = \underset{u}{2\cos x}\underset{v}{(\sin x - 1)}$ $x \in [-\frac{\pi}{2}, \frac{3\pi}{2}]$

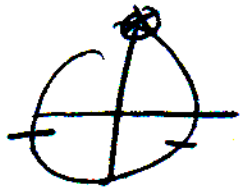
$f'(x) = -2(\sin x - 1)\sin x + 2\cos^2 x$

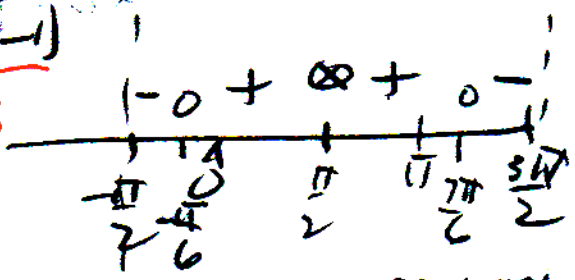
$= -2\sin^2 x - 2\sin x + 2 - 2\sin^2 x$

$= -4\sin^2 x - 2\sin x + 2$

$= -2(2\sin x + 1)(\sin x - 1)$

5pts





$f(-\frac{\pi}{2}) = 0 \cdot (\ ) = 0$

$f(-\frac{\pi}{6}) = 2(\frac{\sqrt{3}}{2})(-\frac{3}{2})$

$f(\frac{7\pi}{6}) = 2(-\frac{\sqrt{3}}{2})(-\frac{3}{2})$

$f(\frac{3\pi}{2}) = 0 \cdot (\ ) = 0$

Rel max dec fn it $(-\frac{\pi}{2}, 0)$ $(\frac{7\pi}{6}, \frac{3\sqrt{3}}{2})$ max Abs f' test

4 pts min f' test $(\frac{\pi}{6}, -\frac{3\sqrt{3}}{2})$ $(\frac{3\pi}{2}, 0)$ min dec to it Rel

II $\underset{u}{x^2}\underset{v}{y} + \sin(\underset{u}{xy}) = \pi$

$2xy + x^2\frac{dy}{dx} + \cos(xy)\left[y + x\frac{dy}{dx}\right] = 0$ 10 pts

$\pi x^2 + 0 = \pi$ $(1, \pi)$

$x^2 = 1$

$x = \pm 1$

$2\pi + \frac{dy}{dx} + (-1)\left[\pi + \frac{dy}{dx}\right] = 0$

$2\pi + \frac{dy}{dx} - \pi - \frac{dy}{dx} = 0$

$\pi = 0$

Reject

$(-1, \pi)$ $-2\pi + \frac{dy}{dx} - [\pi - \frac{dy}{dx}] = 0$

$-2\pi + \frac{dy}{dx} - \pi + \frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{3\pi}{2} = m$ $(-1, \pi) \rightarrow$

$y - y_1 = m(x - x_1)$

$y - \pi = \frac{3\pi}{2}(x + 1)$

$y = \frac{3\pi}{2}x + \frac{5\pi}{2}$

$\frac{3}{2}$ $\frac{4}{3}$ $\frac{9}{6}$ $\frac{8}{6}$

$f(2)=3$	$f'(2)=12$	$g(2)=3$	$g'(2)=-2$
$f(3)=5$	$f'(3)=12$	$g(3)=2$	$g'(3)=-3$

a) $(fg)'\Big|_{x=2}$ b) $\left(\frac{f}{f+g}\right)'\Big|_{x=3}$ c) $(f(g))'\Big|_{x=3}$

$gf' + fg'$

$(3)(12) + 3(-2)$

$36 - 6$

30

$$\frac{(f+g)f' - f(f'+g')}{(f+g)^2}$$

$$\frac{(5+2)12 - 5(12-3)}{(5+2)^2}$$

$$\frac{84-45}{49} = \frac{39}{49}$$

$f'(g)\,g'$

$f'(g(3))\,g'$

$f'(2)\,g'(3)$

$12(-3)$

-36

IV they are correct. we are given a fctn $f(x)$. we compute $f'(x)$. we pick an x_1 guess. the coordinates of the point are $(x_1, f(x_1))$ and the slope of the tangent line is $f'(x_1)$

$y - y_1 = m(x - x_1)$

$y - f(x_1) = f'(x_1)(x - x_1)$ we let $y=0$ to find the x-intercept which will be our next guess.

Extra. $\frac{2(9x-2)}{135}\sqrt{(3x+1)^3}$

$$\frac{18}{135}\sqrt{(3x+1)^3} + \frac{2(9x-2)}{135}\cdot\frac{3}{2}(3x+1)^{\frac{1}{2}}\cdot 3x$$

$$\frac{2}{15}(3x+1)^{\frac{3}{2}} + \frac{9x}{15}(9x-2)(3x+1)^{\frac{1}{2}}$$

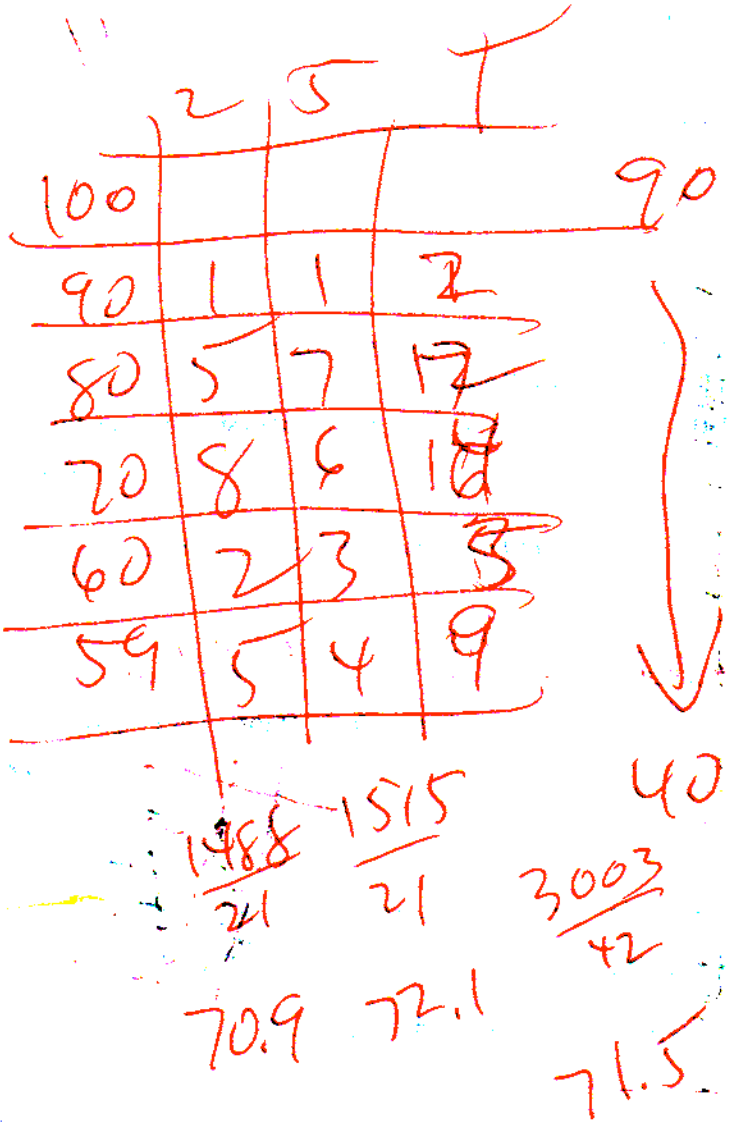
$$\frac{(3x+1)^{\frac{1}{2}}}{15}\left[2(3x+1) + 9x - 2\right] \Rightarrow$$

$6x+2+9x-2$

$15x$

$x(3x+1)^{\frac{1}{2}}$

$x\sqrt{3x+1}$



1488/21 = 70.9

1515/21 = 72.1

3003/42 = 71.5

ALL the "f"s earned the f. rewarding as shown should make this a super exam.

Pblm #I B was pblm 22 which was the demo pblm. for the sections!

Honors Analysis

Name AN L Ch 4 mech

Copy original problem.

Per _____ Date 94

Convince *me* that you understand the concept!

Chapter 4 – Mechanical Exam

Do not use calculators on this exam.

I Find and identify (including "relative" or "absolute") the coordinates of all extrema for each of the following. Remember to consider the domain limits. Do **not** draw the graph of the function. All conclusions must be properly justified. Be *very* sure the reader understands your logic!

(25 pts ea)

a) $f(x) = 2x^3 - 4x^2 \quad x \in [-2, 3]$

b) $f(x) = 2\cos x(\sin x - 1) \quad x \in [-\frac{\pi}{2}, \frac{3\pi}{2}]$

II Given: $x^2y + \sin(xy) = \pi$. Find the equation of the tangent line when $y = \pi$. (hint: There *is* only one such line.)

(20 pts)

III

Given function f for which $f(2) = 3$, $f(3) = 5$, $f'(2) = 12$, $f'(3) = 12$ and g for which $g(2) = 3$, $g(3) = 2$, $g'(2) = -2$, $g'(3) = -3$.

x	2	3
f	3	5
f'	12	12
g	3	2
g'	-2	-3

This means "the derivative of the product of f and g evaluated at x = 2: $(fg)'\big|_{x=2}$"

Evaluate: (20 pts tot)

a) $(fg)'\big|_{x=2}$

b) $\left(\frac{f}{f+g}\right)'\Big|_{x=3}$

c) $(f(g))'\big|_{x=3}$

IV Someone described Newton's method (for finding roots) as "just locating x–intercepts of tangent lines." Explain what they meant without using a specific function.

(10 pts)

Extra Credit ---------- 5 pts ----------

Find $f'(x)$. Simplify completely. $f(x) = \frac{2(9x-2)}{135}\sqrt{(3x+1)^3}$

Make up 2000

	5	6	T
100	0	0	0
90	5	4	9
80	1	4	5
70	5	2	7
60	4	0	4
54	2	1	3

99

26

78.5

1288 / 17 = 75.8

909 / 11 = 82.64

Nate Sverlow 84
Robert Delgato 84
Tracy Bunting 83
corey farnell 78
Dustin Devon 78
ELAINE MAC 74
Julie Wep 73
Robert Green 70
AARON Stogner 35

4/25/2002

Jenna Fussell 84
Rachel Heiz 79

make changes on industrial strength sheet as suggested in notebook. while the mistakes weren't caused by not doing the sheet — they couldn't draw graphs & circles for example — no study skills ability at all.

Honors Analysis

Copy original problem.

Convince *me* that **you** understand the concept!

***Any* decimals must be accurate to 3 places.**

Name ANL ch 4m

Per _____ Date 4/25/95

Chapter 4m Exam

I Find the coordinates of all extreme points. Distinguish between "relative" and "absolute" extrema. Justify. Sketch. (25 pts ea)

a) $f(x) = \sin 2x - 2\sin x \quad -\pi \le x \le \pi$ b) $f(x) = x^3 + 3x^2 - 4 \quad -4 \le x \le 2$

II Given the circle $(x+1)^2 + (y-2)^2 = 25$ and the line $x = 3$. (30 pts tot)

a) Sketch the circle and the line on the same axis. Find their coordinates of intersection.

b) Find $\frac{dy}{dx}$ at each of the points where the line and the circle intersect.

c) A line which is perpendicular to another line at a specific point is said to be the "normal line" at the point. A normal line to a curve is the line perpendicular to the tangent line to the curve at the point. Find the equations of the "normal lines" to the circle at each of the points found above.

d) At what coordinate do the two normal lines intersect?

III Given $f(x)$. Determine $f'(x)$. Do not simplify your answers. (for example, leave negative exponents as negative exponents, do not combine constants, etc.) Basically, I am looking for evidence that you have used the proper formular at the proper time. (10 pts)

$$f(x) = \frac{\left(2x^3 + 3\right)^5 \left(3 - 2\sqrt{x}\right)}{(5x + 3)^2}$$

IV There are six basic trig functions. List each along with its derivative. (10 pts tot)

Extra Credit ---------- 5 pts ----------

Given $f(x) = 3\cos 2x + (1 - \sin x)$ and $g(x) = 3\cos|2x| + |\sin x - 1|$.

Explain why the graph of $f(x)$ is identical to the graph of $g(x)$.

I a) $f(x) = \sin 2x - 2\sin x$

$-\pi \le x \le \pi$

$f'(x) = 2\cos 2x - 2\cos x$

$2(\cos^2 x - \sin^2 x - \cos x)$

$2(2\cos^2 x - \cos x - 1)$

$2(2\cos x + 1)(\cos x - 1)$

$\cos x = -\frac{1}{2} \quad \cos x = 1$

$x \in \{-\frac{2\pi}{3}, \frac{2\pi}{3}\}$ 3 pts

$2(1)(-4) < 0$

f': $+\ 0\ -\ 0\ -\ 0\ +$

$-\pi \quad -\frac{2\pi}{3} \quad 0 \quad \frac{\pi}{2} \quad \frac{2\pi}{3} \quad \pi$

$f(-\pi) = 0$ $(-\pi, 0)$ min endpt inc fn'd Rel

$f(-\frac{2\pi}{3}) = \frac{\sqrt{3}}{2} + \sqrt{3} = \frac{3\sqrt{3}}{2}$ $(-\frac{2\pi}{3}, \frac{3\sqrt{3}}{2})$ max f'tes Abs

$f(0) = 0$ $(0,0)$ LPI

$f(\frac{2\pi}{3}) = -\frac{\sqrt{3}}{2} - \sqrt{3} = -\frac{3\sqrt{3}}{2}$ $(\frac{2\pi}{3}, -\frac{3\sqrt{3}}{2})$ min f'tes Abs

$f(\pi) = 0$ $(\pi, 0)$ endpt max. inc to it rel

b) $f(x) = x^3 + 3x^2 - 4 \quad -4 \le x \le 2$

$f'(x) = 3x^2 + 6x$

$3x(x+2)$

$+\ 0\ -\ 0\ +$ f'

$-4 \quad -2 \quad 0 \quad 2$

$-\ 0\ +$ f''

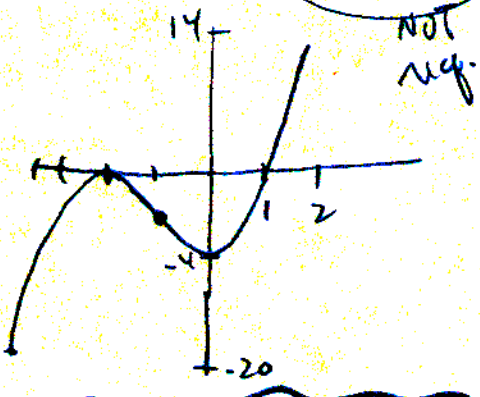
-1

	1	3	0	−4	
−4	1	−1	4	−20	(−4,−20) min endpt Abs inc fn'd
−2	1	1	−2	0	
0				−4	(−2,0) max f'tes Rel
2	1	5	10	16	(0,−4) min f'tes Rel
−1	1	2	−2	−2	(2,16) max endpt Abs, inc to it

$f''(x) = 6x + 6$

$6(x+1)$

$(-1,-1)$ Actual PI



NOT neg.

II a) $(x+1)^2 + (y-2)^2 = 25$

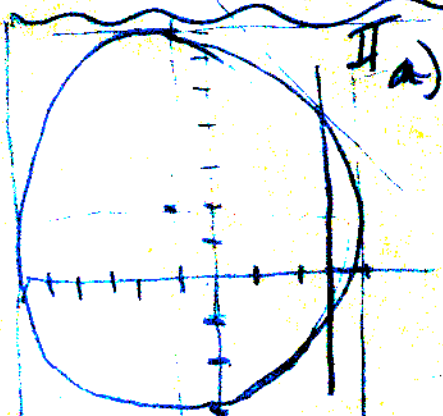
$4^2 + (y-2)^2 = 25$

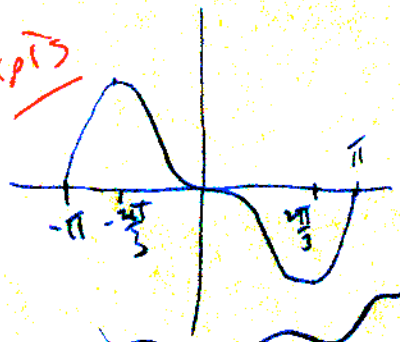
$(y-2)^2 = 9$

$y - 2 = \pm 3$

$y \in \{-1, 5\}$

$(3,-1)$ $(3,5)$





b) $\frac{dy}{dx}\Big|_{(3,5)} = -\frac{4}{3} \perp m = \frac{3}{4}$

$y - 5 = \frac{3}{4}(x-3)$

$y = \frac{3}{4}x - \frac{9}{4} + \frac{20}{4}$

$y = \frac{3}{4}x + \frac{11}{4}$

$\frac{dy}{dx} = -\frac{(x+1)}{y-2}$

$\frac{dy}{dx}\Big|_{(3,-1)} = \frac{-4}{-3} = \frac{4}{3} \perp m = -\frac{3}{4}$

$y + 1 = -\frac{3}{4}(x-3)$

$y = -\frac{3}{4}x + \frac{9}{4} - \frac{4}{4}$

$y = -\frac{3}{4}x + \frac{5}{4}$

$\frac{3}{4}x + \frac{11}{4} \overset{\text{set}}{=} -\frac{3}{4}x + \frac{5}{4}$

$\frac{6}{4} = -\frac{6}{4}x$ ⊥ lines intersect

$x = -1 \quad y = 2$ at $(-1, 2)$

III

$$f(x) = \frac{(2x^3+3)^5 (3-2\sqrt{x})}{(5x+3)^2}$$

$$\frac{(5x+3)^2\left[(3-2\sqrt{x})\,5(2x^3+3)^4 6x^2 + (2x^3+3)^5\left(-\frac{2}{2\sqrt{x}}\right)\right] - (2x^3+3)^5(3-2\sqrt{x})\,2(5x+3)5}{\left((5x+3)^2\right)^2}$$

IV

$A(x) = \sin x$
$A' = \cos x$

$B(x) = \cos x$
$B' = -\sin x$

$C(x) = \tan x$
$C' = \sec^2 x$

$D(x) = \csc(x)$
$D'(x) = -\csc x \cot x$

$E(x) = \sec x$
$E'(x) = \sec x \tan x$

$f(x) = \tan x$
$f'(x) = \sec^2 x$

EXTRA

$f(x) = 3\cos 2x + (1 - \sin x)$

$g(x) = 3\cos|2x| + |\sin x - 1|$

2 Terms.

(1) Cos is even so $\cos(-2x) = \cos 2x$ so the sign of x has no effect.

(2) $\sin x - 1$ is ≤ 0 !,

$-(\sin x - 1)$ is from Def of abs val

$+(1 - \sin x)$.

So f and g are identical so they should have the same graph.

Honors Analysis

Copy original problem.

Convince *me* that **you** understand the concept!

No decimal answers, please.

Name Anl Ch 4m #2

Per _____ Date 5/14/96

Became 99 Make up 3rd Am

Chapter 4m Exam (again)

I Given $f(x) = \dfrac{3x(2x-1)}{5x-2}$. Determine and identify the (x, y) coordinates of all extrema between $x = -2$ and $x = 2$ inclusive. Justify, of course. (25 pts)

II Given the curve: $3x^2 - 2xy - y^2 = -9$ and the line $x = 2$. (30 pts tot)

a) Find the coordinates of intersection of the line and the curve.

b) Find the equation of each of the tangent lines to the curve at those points.

c) Find the coordinates of the intersection of the tangent lines.

III Show that for any real number a, the recursion formula of Newton's Method for approximating $\sqrt[4]{a}$ can be written in the form: $\dfrac{1}{4}\left(3x_n + \dfrac{a}{x_n^3}\right)$ (25 pts)

IV Given $f(x)$. Determine $f'(x)$. Do not simplify your answers. (For example, leave negative exponents as negative exponents, do not combine constants, etc.) I am looking for evidence that you have used the proper derivative formula at the proper time. (20 pts)

$$f(x) = \left(\frac{2\sin x \cos x}{\cos^2 x - \sin^2 x}\right)^3$$

Extra Credit ---------- **5 pts** ----------

Referring to $f(x)$ in Section IV above, find $f'\left(\frac{\pi}{6}\right)$.

I

$$f(x) = \frac{3x(2x-1)^4}{(5x+2)^5} \quad x \in [-2, 2]$$

$$f'(x) = \frac{(5x+2)^5[(2x-1)3 + 3x(2)] - 3x(2x-1)^4 5}{(5x+2)^{10}}$$

$$(5x+2)(6x-3+6x) - 15x(2x-1)$$

$$3(5x+2)(4x-1) - 15x(2x-1)$$

$$3[20x^2+3x-2-10x^2+5x]$$

$$6[5x^2+4x-1]$$

$$\frac{6(5x-1)(x+1)}{(5x+2)^6}$$

Note. $x = -\frac{2}{5}$ is non-reducible domain restriction

f': $+$ 0 $-$ $\nexists$ $-$ 0 $+$ on $[-2, -1, -\frac{2}{5}, 0, \frac{1}{5}, 2]$

$f(-2) = \frac{-3(-5)^4}{-84} = -\frac{15}{4}$

$f(-1) = \frac{-3(-1)^4}{(-3)^5} = -3$

$f(\frac{1}{5}) = \frac{3(\frac{1}{5})^4(-\frac{3}{5})}{(\frac{1}{5}+2)^5} = -\frac{3}{25}$

$f(2) = \frac{6(3)^4}{12^5} = \frac{3}{2}$

$(-2, -\frac{15}{4})$ min inc to it

$(-1, -3)$ max f' to it.

$(\frac{1}{5}, -\frac{3}{25})$ min f' to it

$(2, \frac{3}{2})$ max inc to it

II $3x^2 - 2xy - y^2 = -9$

a) $x = 2$ $\quad 12 - 4y - y^2 = -9$

$y^2 + 4y - 21 = 0$

$(y-3)(y+7) = 0$

$(2, 3)$

$(2, -7)$

b) $6x - (2y + 2x\frac{dy}{dx}) - 2y\frac{dy}{dx} = 0$

$6x - 2y - 2x\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$

$(2x+2y)\frac{dy}{dx} = 6x - 2y$

$\frac{dy}{dx} = \frac{3x-y}{x+y}$

$(2, 3)$ $\quad \frac{6-3}{2+3} = \frac{3}{5} = m$

$y - 3 = \frac{3}{5}(x-2)$

$y = \frac{3}{5}x + \frac{9}{5}$

$(2, -7)$ $\quad \frac{6+7}{2-7} = \frac{13}{-5} = m$

$y + 7 = -\frac{13}{5}(x-2)$

$= -\frac{13}{5}x + \frac{26}{5} - \frac{35}{5} = -\frac{13}{5}x - \frac{9}{5}$

c) $y = \frac{3}{5}x + \frac{9}{5}$

$y = -\frac{13}{5}x - \frac{9}{5}$

intersect at

$\frac{3}{5}x + \frac{9}{5} = -\frac{13}{5}x - \frac{9}{5}$

$\frac{16}{5}x = -\frac{18}{5}$

$x = -\frac{9}{8}$

$y = \frac{3}{5}(-\frac{9}{8}) + \frac{9}{5} = \frac{-27+72}{40} = \frac{45}{40} = \frac{9}{8}$

$(-\frac{9}{8}, \frac{9}{8})$

III $\sqrt[4]{a}$

Let $x = \sqrt[4]{a}$

$x^4 = a$

$x^4 - a = 0$

$f(x) = x^4 - a$

$f'(x) = 4x^3$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$= x_n - \frac{(x_n^4 - a)}{4x_n^3}$

$= \frac{4x_n^4 - x_n^4 + a}{4x_n^3}$

$= \frac{3x_n^4 + a}{4x_n^3}$

$\frac{1}{4}\left[\frac{3x_n^4}{x_n^3} + \frac{a}{x_n^3}\right]$

$\frac{1}{4}\left[3x_n - \frac{a}{x_n^3}\right]$

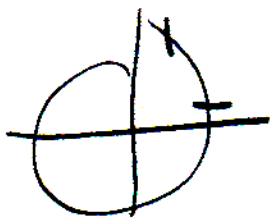
IV $f(x) = \left(\frac{2\sin x \cos x}{\cos^2 x - \sin^2 x}\right)^3$

$f'(x) = 3\left(\frac{2\sin x \cos x}{\cos^2 x - \sin^2 x}\right)^2 \left[(\cos^2 x - \sin^2 x)\left[2\cos^2 x + 2\sin x(-\sin x)\right] - \cdots\right.$

$\left. \cdots 2\sin x \cos x \left[2\cos x(-\sin x) - 2\sin x(\cos x)\right]\right.$

$\left. \over (\cos^2 x - \sin^2 x)^2 \right.$

Extra $f(x) = \left(\frac{2\sin x \cos x}{\cos^2 x - \sin^2 x}\right)^3$



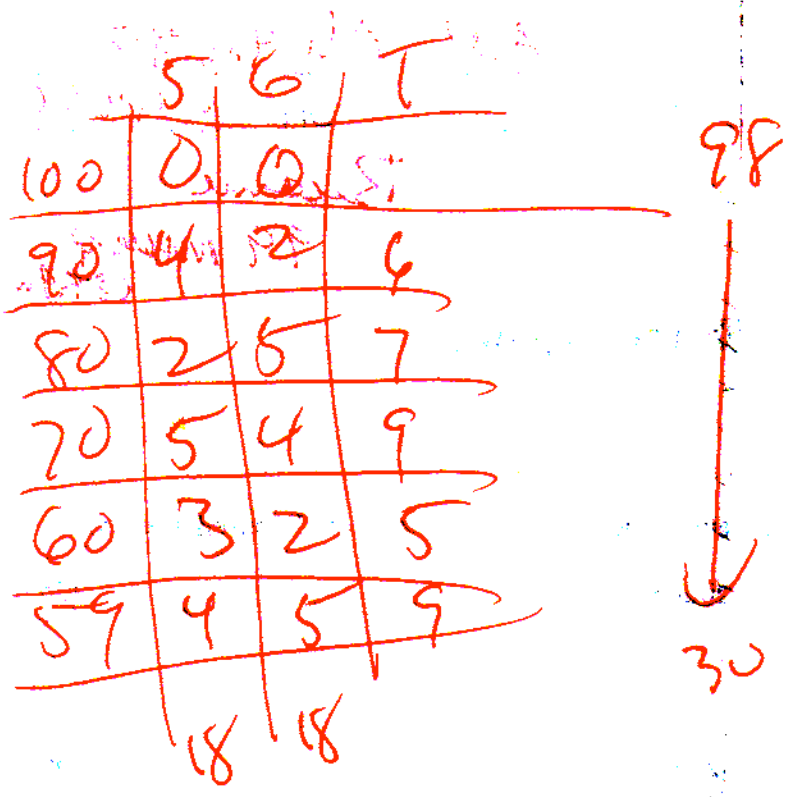
$= \left(\frac{\sin 2x}{\cos 2x}\right)^3$

$= \tan^3 2x$

$f'(x) = 3\tan^2 2x (\sec^2 2x)\, 2$

$f'\left(\frac{\pi}{6}\right) = 3 \cdot 3 \cdot 4 \cdot 2$

$72.$



1999

Renata Brodsky 88
Barrett Warn 42
Julia Beckham 66
Tara Holstein 55
Jason Rachel 56

2001

Wm King-Lewis 83
Jimmy ALonso 57
Stephanie Kuntz 54
Sunday Sawkaran 30

1316/18 1280/18 2596/36

73.1 71.1 72.1

One typo on 1st problem actually made the problem easier.

Question III was almost an exact copy of a lecture problem AND a HW problem

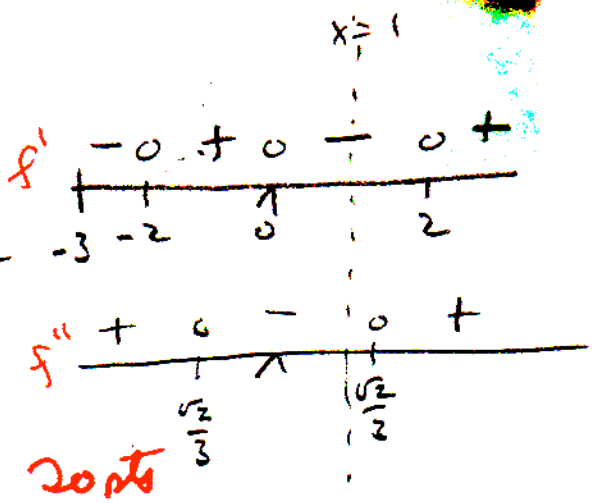
5 A students in pre-S and 2 A students in pre-S did not take exam.

I $f(x) = x^4 - 8x^2 + 8$ $[-3, 1]$

4)

$f'(x) = 4x^3 - 16x = 4x(x^2 - 4)$

$4x(x-1)(x+2)$



20 pts

$f''(x) = 12x^2 - 16$

$4(3x^2 - 4)$

$f =$	1	0	−8	0	8
−3	1	−3	1	−3	17
−2	1	−2	−4	8	−8
0					8
1	1	1	−7	0	1

$(-3, 17)$ Abs max "decrease from it"

$(-2, -8)$ Abs min f'' test

$(0, 8)$ rel max f'' test

$(1, 1)$ rel min "decrease to it"

3) $f(x) = \sin 2x - 2\sin x$ $[0, 2\pi]$

$f'(x) = 2\cos 2x - 2\cos x$

$2(\cos^2 x - \sin^2 x) - 2\cos x$

$2(2\cos^2 x - 1) - 2\cos x$

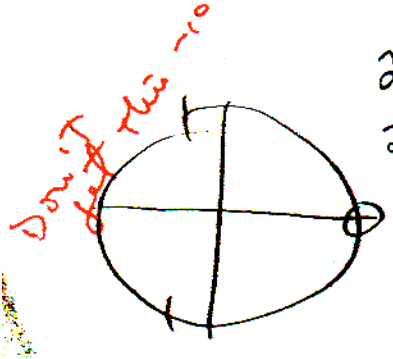
$4\cos^2 x - 2\cos x - 2$

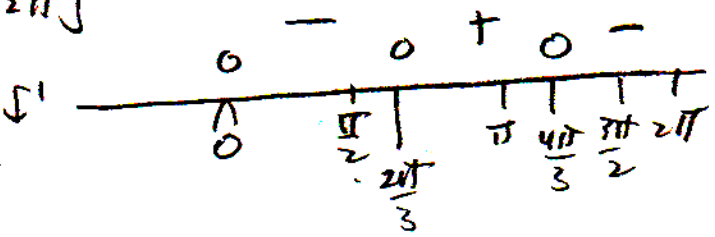
$2(2\cos^2 x - \cos x - 1)$

$2(2\cos x + 1)(\cos x - 1) \overset{\text{set}}{=} 0$

$\cos x = -\frac{1}{2}$ $\cos x = 1$

$x = \{0, \frac{2\pi}{3}, \frac{4\pi}{3}\}$





$f'(\frac{\pi}{2}) = 2\cdot(-1) - 0 = -2$

$f'(\pi) = 2(1) + 2 = 4$

$f'(\frac{3\pi}{2}) = -2 - 0 = -2$

rel

$f(0) = 0$ $(0, 0)$ MAX "decrease from it"

$f(\frac{2\pi}{3}) = \sin\frac{4\pi}{3} - 2\sin\frac{2\pi}{3} = -\frac{\sqrt{3}}{2} - \sqrt{3} = -\frac{3\sqrt{3}}{2}$

$(\frac{2\pi}{3}, -\frac{3\sqrt{3}}{2})$ ABS min f'' test

$f(\frac{4\pi}{3}) = +\frac{\sqrt{3}}{2} + \sqrt{3}$ ABS

$(\frac{4\pi}{3}, +\frac{3\sqrt{3}}{2})$ max f'' test

20 pts

$f''(x) = -4\sin 2x + 2\sin x$

$f''(0) = 0$ f'' test fail

$f''(\frac{2\pi}{3}) = -4\sin\frac{4\pi}{3} + 2\sin\frac{2\pi}{3} = \frac{4\sqrt{3}}{2} + \frac{2\sqrt{3}}{2} > 0$ min f'' test

$f''(\frac{4\pi}{3}) = -4\sin\frac{8\pi}{3} + 2\sin\frac{4\pi}{3} = -\frac{4\sqrt{3}}{2} - \frac{4\sqrt{2}}{2} < 0$ max f'' test

$f(2\pi) = f(0) = 0$ "decrease to it"

∴ min (rel)

II

4) $\dfrac{(2x+1)^5(4x-5)^6}{(7x-8)^{10}}$ u v — 15 pts

$$\frac{(7x-8)^{10}\left[(4x-5)^6 5(2x+1)^4 2 + (2x+1)^5 6(4x-5)^5 4\right] - (2x+1)^5(4x-5)^6 10(7x-8)^9 7}{(7x-8)^{20}}$$

B) $5x^2(1+\sqrt{3x})^4$ u v

$(1+\sqrt{3x})^4 10x + 5x^2 4(1+\sqrt{3x})^3 \frac{3}{2\sqrt{3x}}$ — 15 pts

III

an x value within an interval as determined by critical values near the root is used to get the equation of the tangent line. This line's x intercept is the next guess, the procedure is repeated until desired accuracy is achieved the "formula" which actually "picks" the values is

$$x_{N+1} = x_N - \frac{f(x_N)}{f'(x_N)}$$

10 pts

IV

A) $x^3+3xy+y^3=1$ $(2,-1)$

$3x^2 + 3y + 3x\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$ — 3 pts.

$$\frac{dy}{dx} = -\frac{(x^2+y)}{x+y^2}\Bigg|_{2,-1}$$

$$= -\frac{4-1}{2+1}$$

$$= -\frac{3}{3}$$

$$= -1$$ — 6

$y+1 = -(x-2)$

$1 = -x+2$

$x = 1$ 10 pts

B) $y = \sqrt{5^2-x^2}$ $(3,4)$

$$\frac{dy}{dx} = \frac{-x}{\sqrt{5^2-x^2}}\Bigg|_{3,4} = -\frac{3}{4}$$

$y-4 = -\frac{3}{4}(x-3)$ — 6

$-4 = -\frac{3}{4}(x-3)$

$\frac{16}{3} = x-3$

$x = \frac{25}{3}$ 10 pts

Extra

$B(x)=\cos x$	$B'(x)=-\sin x$
$C(x)=\tan x$	$C'(x)=\sec^2 x$
$D(x)=\sec x$	$D'(x)=\sec x\tan x$
$E(x)=\csc x$	$E'(x)=-\csc x\cot x$
$F(x)=\cot x$	$F'(x)=-\csc^2 x$

92

	2	5	T
100	0	4	
90	2	5	
80	5	8	
70	4	1	
60	0	1	
59	2	2	

$\frac{1004}{13}$ $\frac{1795}{21}$ $\frac{2799}{34}$

77.23 85.5 82.3

~~8211~~

change limits on Ib to $-\frac{\pi}{2} \le x \le \frac{11\pi}{6}$
so PI # 0 enters the picture.

EASY, excellent exam.

97

100	0
90	4
80	9
70	4
60	6
59	2

98 → 53

1958 / 25 = 78.5

Still on easy test — Lousy students.

most low scores were re
who wasted time on
when it wasn't the
they didn't read the

86

excellent exam

	5	2	T	
100	0	0	0	99
90	4	4	8	
80	4	8	12	
70	4	3	7	
60	3	4	7	
59	7	4	11	18
	71.6	77.0	74.3	

Honors Analysis
Copy original problem.
Convince *me* that **you** understand the concept!

Name _____ 97

Per _____ Date _____

Chapter 4 "Mechanical" Exam

I Find and identify all extrema for each of the following. Each extrema should be labeled as "global" or "relative" in addition to max or min. Remember, end points yield extrema! For all extrema, you must justify (using a proper statement). (20 pts ea)

a) $f(x)=x^4-8x^2+8 \;;\; x \in [-3,1]$

b) $f(x)=\sin 2x-2\sin x \;;\; x \in [0,2\pi]$

II Given $f(x)$, determine $f'(x)$. DO NOT SIMPLIFY! Basically, I am looking to see that you are using the appropriate formula properly. I am not testing your ability to use the distributive property. (15 pts ea)

a) $\dfrac{(2x+1)^5(4x-5)^6}{(7x-8)^{10}}$

b) $5x^2\left(1+\sqrt{3x}\right)^4$

III Explain "Newton's Method" for finding roots of equations. Do not use specific examples. Conclude your explanation with Newton's recursion formula. (10 pts)

IV Determine the x-intercept of the tangent line to the given item at the given point: (10 pts ea)

a) $x^3+3xy+y^3=1 \;;\; (2,-1)$

b) $y=\sqrt{5^2-x^2} \;;\; (3,4)$

Extra Credit 5 pts

There are six basic trig functions.

For example, $A(x)=\sin x$ so $A'(x)=\cos x$.

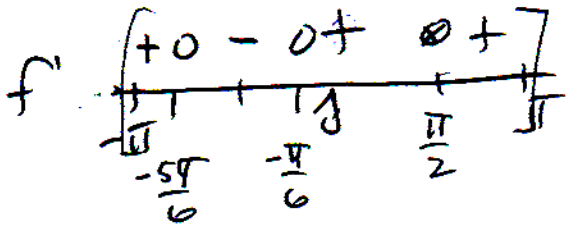
List the other five trig functions and their derivatives in a like manner.

I A) $f(x) = \sin 2x - 2\cos x \quad -\pi \le x \le \pi$

$f'(x) = 2\sin x\cos x - 2\cos x$

$2\cos x(\sin x - 1)$

$u \quad v$



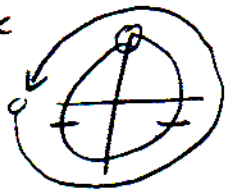
$f'(x) = (\sin x - 1)(-2\sin x) + 2\cos^2 x$

$-2\sin^2 x + 2\sin x + 2 - 2\sin^2 x$

$f'(0) = \quad -4\sin^2 x + 2\sin x + 2$

$-2(2\sin^2 x - \sin x - 1)$ Set

$-2(2\sin x + 1)(\sin x - 1) = 0$



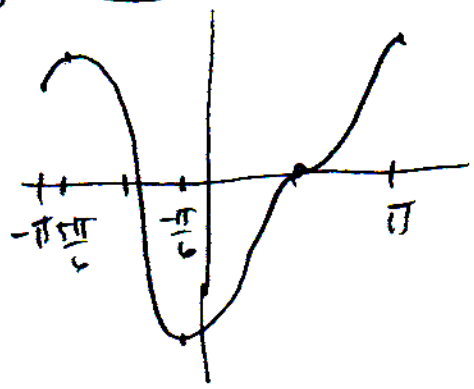
$f'(0) = -2(1)(-1) = +2$

Rel $(-\pi, 2)$ min ENDPT INC fn it.

Abs $(-\frac{5\pi}{6}, 2.598)$ max f' test

Abs $(-\frac{\pi}{6}, -2.598)$ min f' test

Rel $(\pi, 2)$ max ENDPT INC to it.



B) $f(x) = 3x^4 + 8x^3 - 6x^2 - 24x$

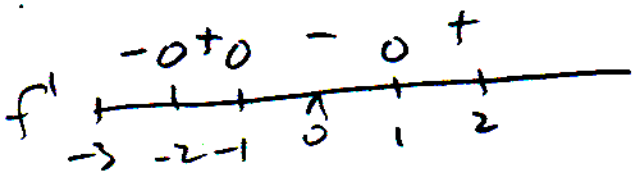
$f'(x) = 12x^3 + 24x^2 - 12x - 24$

$12(x^3 + 2x^3 - x - 2)$

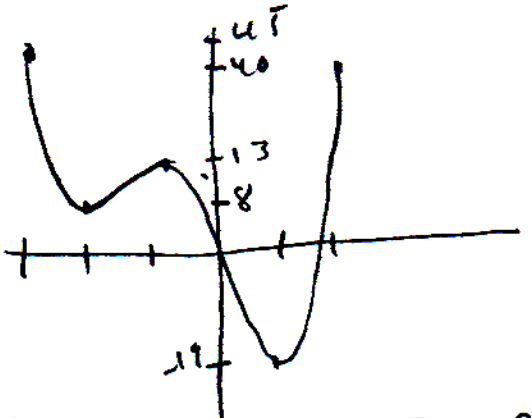
$12(x^2(x+2) - (x+2))$

$12(x^2 - 1)(x+2)$

$12(x-1)(x+1)(x+2)$



	3	8	−6	−24	0
−3	3	−1	−3	−15	45
−2	3	2	−10	−4	8
−1	3	5	−11	−13	13
1	3	11	5	−19	−19
2	3	14	22	20	40



(−3, 45) ABS max ENDPT dec fn it

(−2, 8) rel min f' test

(−1, 13) rel max f' test

(1, −19) ABS min f' test

(2, 40) rel MAX ENDPT INC to it.

II $\sin(x-y) = y\cos x$ $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ 5 pts

$\cos(x-y)[1-y'] = \cos x\, y' + y\sin x$

$\cos[0][1-y'] = 0\cdot y' - \frac{\pi}{2}\cdot 1$

$1 - y' = -\frac{\pi}{2}$

$y' = 1 + \frac{\pi}{2}$

$y - \left(\frac{\pi}{2}\right) = \left(1+\frac{\pi}{2}\right)\left(x - \frac{\pi}{2}\right)$

$y - \frac{\pi}{2} = \left(1+\frac{\pi}{2}\right)x - \frac{\pi}{2} - \frac{\pi^2}{4}$

$y = \left(1+\frac{\pi}{2}\right)x - \frac{\pi^2}{4}$

III $(x_1, f(x_1))$

a) $y - f(x_1) = f'(x)[x - x_1]$

$= x f'(x) - x_1 f'(x_1)$

$y = x f'(x) + f(x_1) - x_1 f'(x_1)$

b) $0 \overset{\text{set}}{=} x f'(x) + f(x_1) - x_1 f'(x_1)$

$x f'(x) = x_1 f'(x_1) + f(x_1)$

$x = x_1 + \frac{f(x_1)}{f'(x_1)}$

EXTRA. $f(x) = x^3 - 3x - 1$

$f'(x) = 3x^2 - 3$

$3(x^2-1)$

$x_1 = 2$

$f(2) = 8 - 7 = 1$

$f'(2) = 9$

$x_2 = 2 + \frac{1}{9}$

$x_2 = \frac{17}{9} \approx 1.889$

$f\left(\frac{17}{9}\right) = .0727$

$f'\left(\frac{17}{9}\right) = 7.7037$

$x_3 = 1.889 - \frac{.0727}{7.7037}$

$x_3 = 1.879$

III $\left[(2x^3+3)^3(3\sin^2 x+1)^2\right]\left[(5-3\sqrt{x})^{5/4}\frac{1}{2}(3-4x^3)^{-\frac{1}{2}}(-12x^2) + \sqrt{3-4x^3}\,\frac{3}{2}(5-3\sqrt{x})^{\frac{1}{2}}\left(-\frac{3}{2}x^{-\frac{1}{2}}\right)\right] - \cdots$

$\cdots \left[\sqrt{3-4x^3}\,(5-3\sqrt{x})^{\frac{1}{2}}\right]\left[(3\sin^2 x+1)^2 3(2x^3+1)^2 6x^2 + (2x^3+3)^3 2(3\sin^2 x+1)(6\sin x)(\cos x)\right]$

$\left[(2x^3+3)^3(3\sin^2 x+1)^2\right]^2$

98

100	0	
90	1	
80	3	
70	6	
60	4	
59	6	

9

21

1462 27

96

	5	6	7
100	4	2	
90	2	0	
80	2	2	
70	2	2	
60	5	2	
59	9	13	22

105

21

1679 1230
24 21

69.96 58.57 64.

Vacation took its toll. most didn't do a good job of D.T.T.O.T.W and we did really have time to go over them in detail Probably need to add a couple of days & some Quizzes.

Honors Analysis

Name Ava Chm

Copy original problem.

Per _____ Date 4/24/98

Convince *me* that **you** understand the concept!

96

(Decimals must be accurate to 3 places.)

Chapter 4m Exam

I Given function and its domain, (25 pts ea)

i. State the coordinates of all maximum and all minimum points. Identify them as relative or ablsolute. Justify, of course.

ii. Sketch $f(x)$. Be sure to indicate all points listed in part "i" above.

a) $f(x) = \sin 2x - 2\cos x$, $x \in [-\pi, \pi]$

b) $f(x) = 3x^4 + 8x^3 - 6x^2 - 24x$, $x \in [-3, 2]$

II Given $\sin(x - y) = y\cos x$. Find the equation of the tangent line to the curve at $(\pi/2, \pi/2)$ (20 pts)

No decimals on this problem. (hint: Do *not* use the sine add/subtract formula.)

III Given a function f and a number x_1. (15 pts total)

a) Find the equation of the tangent line to f at $x = x_1$ (in graphing form).

b) Find the x-intercept of the answer to part a.

IV Given $f(x)$. Determine $f'(x)$. Do not simplify your answers. (For example, leave negative exponents as negative exponents, do not combine constants, etc.) I am looking for evidence that you have used the proper formula at the proper time. (15 pts)

$$f(x) = \frac{\sqrt{3 - 4x^3}\left(5 - 3\sqrt{x}\right)^{\frac{3}{4}}}{\left(2x^3 + 3\right)^3\left(3\sin^2 x + 1\right)^2}$$

Extra Credit ---------- 5 pts ----------

Given $f(x) = x^3 - 3x - 1$.

Use Newton's method for solving equations on f with 2 as your first guess.

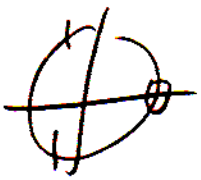
What is your third guess?

I

a) $f(x) = \sin 2x - 2\sin x \quad -\pi < x \le \pi$

$f'(x) = 2\cos 2x - 2\cos x \overset{\text{set}}{=} 0$

$\cos^2 x - \sin^2 x - \cos x = 0$



$2\cos^2 x - \cos x - 1 = 0$

$(2\cos x + 1)(\cos x - 1)$

$x = \{-\frac{2\pi}{3}, 0, \frac{2\pi}{3}\}$

	$-\pi$		$-\frac{2\pi}{3}$		0		$\frac{2\pi}{3}$		π
f'		$+$	0	$-$	$\not\exists$	$-$	0	$+$	

$f(\frac{\pi}{2}) = 0 - 2 = -2$

$f(-\pi) = 0$

$f(-\frac{2\pi}{3}) = \sin\frac{-4\pi}{3} - 2\sin(\frac{-2\pi}{3})$

$\frac{\sqrt{3}}{2} - 2(-\frac{\sqrt{3}}{2}) = \frac{3\sqrt{3}}{2}$

$f(0) = 0$

$f(\frac{2\pi}{3}) = \sin\frac{4\pi}{3} - 2\sin\frac{2\pi}{3}$

$-\frac{\sqrt{3}}{2} - 2(\frac{\sqrt{3}}{2}) = -\frac{3\sqrt{3}}{2}$

$f(\pi) = 0$

rel min $(-\pi, 0)$ ENDPT INC fun'l

abs max $(\frac{2\pi}{3}, \frac{3\sqrt{3}}{2})$ f'l test

$(0,0)$ LPI up → dn

abs min $(\frac{2\pi}{3}, -\frac{3\sqrt{3}}{2})$ f'l test

rel max $(\pi, 0)$ ENDPT INC to it.

B) $f(x) = x^3 + 3x^2 - 4 \quad -4 \le x \le 2$

$f'(x) = 3x^2 + 6x$

$3x(x+2)$

	-4		-2		0
f'	$+$	0	$-$	0	$+$

$f''(x) = 6x + 6$

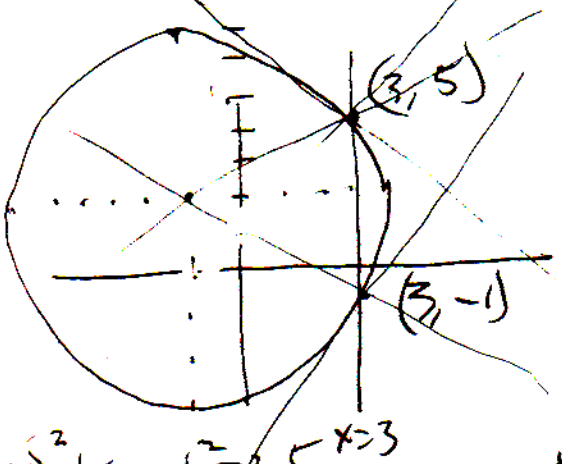
$6(x+1)$

	-4		-1		0
f''	$-$	0	$+$		

	1	3	0	-4	
-4	1	-1	4	-20	$(-4,-20)$ abs min END PT INC fun'l
-2	1	1	-2	0	$(-2,0)$ max f'l Local
-1	1	2	-2	-2	
0				-4	$(0,-4)$ local min f'l
2	1	5	10	16	$(2,16)$ abs max END PT INC to it.

II

a)



$(x+1)^2 + (y-2)^2 = 25$ $x=3$

$16 + (y-2)^2 = 25$

$(y-2)^2 = 9$

$y - 2 = \pm 3$

$y = \{5, -1\}$

b)

$2(x+1) + 2(y-2)\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{(x+1)}{y-2}\Big| = -\frac{(4)}{-3} = \frac{4}{3}$

$(3,-1)$

$\Big| = \frac{-4}{-3} = -\frac{4}{3}$

$(3,5)$

c) $m = -\frac{3}{4}$ $(3,-1)$

$y + 1 = -\frac{3}{4}(x-3)$

$y = -\frac{3}{4}x + \frac{9}{4} - \frac{4}{4}$

$y = -\frac{3}{4}x + \frac{5}{4}$

$m = \frac{3}{4}$ $(3,5)$

$y - 5 = \frac{3}{4}(x-3)$

$= \frac{3}{4}x - \frac{9}{4} + \frac{20}{4}$

$y = \frac{3}{4}x + \frac{11}{4}$

III a) $x^2 + y^2 + 2x - 4y - 20 = 0$

II d $y = -\frac{3}{4}x + \frac{5}{4} \overset{SJ}{=} \frac{3}{4}x + \frac{11}{4} = 7$

$-\frac{6}{4} = \frac{3}{2}x$ The lines intersect at

$-\frac{3}{2} = \frac{3}{2}x$ $(-1, 2)$

$x = -1$

$y = -\frac{3}{4}(-1) + \frac{5}{4}$

$y = 2$

III $f(x) = \frac{(2x^3+3)^5 (3-2\sqrt{x})^4}{(5x+3)^2}$

$f'(x) = \frac{(5x+3)^2\left[(3-2\sqrt{x})\,5(2x^3+3)^4 6x^2 + (2x^3+3)\left(-\frac{2}{2\sqrt{x}}\right)\right] - (2x^3+3)(3-2\sqrt{x})\,2(5x+3)(5)}{((5x+3)^2)^2}$

Honors Analysis

Name _____

Copy original problem.

Per _____ Date _____

Convince *me* that **you** understand the concept*!*

No Calculators!

Chapter 4m Make Up Exam

I Find the coordinates of all extreme points. Distinguish between "relative" and "absolute" extrema. Justify properly. Sketch. (25 pts ea)

a) $f(x) = \sin 2x - 2\sin x \qquad -\pi \le x \le \pi$

b) $f(x) = x^3 + 3x^2 - 4 \qquad -4 \le x \le 2$

II Given the circle $(x+1)^2 + (y-2)^2 = 25$ and the line $x = 3$. (35 pts tot)

a) Sketch the circle and the line on the same axis. Find their coordinates of intersection.

b) Find $\frac{dy}{dx}$ at each of the points where the line and the circle intersect.

c) A line which is perpenducilar to another line at a specific point is said to be the "normal line" at the point. A normal line to a curve is the line perpendicular to the tangent line to the curve at the point. Find the equations of the "normal lines" to the circle at each of the points found above.

d) At what coordinate do the two normal lines intersect?

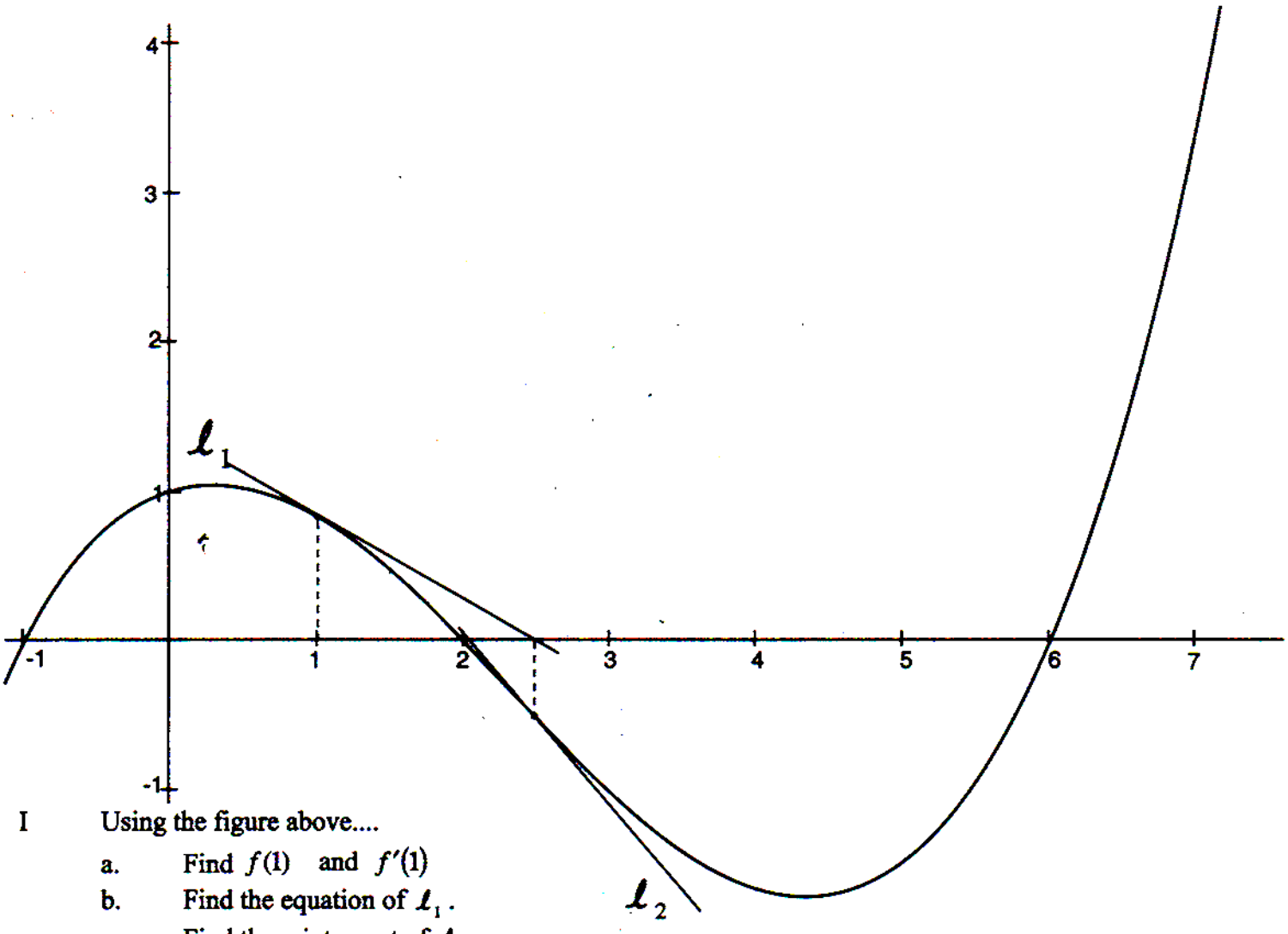
III Given $f(x)$. Determine $f'(x)$. Do not simplify your answers. (for example, leave negative exponents as negative exponents, do not combine constants, etc.) Basically, I am looking for evidence that you have used the proper formular at the proper time. (15 pts)

$$f(x) = \frac{(2x^3+3)^5(3-2\sqrt{x})}{(5x+3)^2}$$

Honors Analysis **Newton Searches for Easter Eggs** Name _____

Mr. Mumaugh Per _____

Given: $f(x)=\left(\frac{1}{12}\right)(x^3-7x^2+4x+12)$



I Using the figure above....

a. Find $f(1)$ and $f'(1)$

b. Find the equation of ℓ_1.

c. Find the x-intercept of ℓ_1.

d. The figure has a dotted line from your answer to part c to the graph of f. Using your calculator, find $f(2.4286)$ and $f'(2.4286)$.

e. Compute the x-intercept of ℓ_2. That is the third guess. (our first guess was 1, our second guess was 2.4286) _____

f. Compute the 4th guess._____

II Do the same thing as we did in section I while seeking the root at $x=6$. Use $x=5$ as your first guess. Draw the lines (for 1st, 2nd, and 3rd guesses). Label them including the x-intercepts. Fill in the table:

1^{st} *guess* → 5	4^{th} *guess* → _____
2^{nd} *guess* → _____	5^{th} *guess* → _____
3^{rd} *guess* → _____	6^{th} *guess* → _____

Let $f(x)$ be the function for which we are seeking the x-intercept. (ie That would be the zero of the function). It follows that $f'(x)$ would be the derivative of f. (Otherwise known as the slope of the tangent line to the function at the value x.)

Let the first guess be x_1.

The coordinate $(x_1, f(x_1))$ is the point on the curve and the tangent line at that point has slope $f'(x)$.

Write the equation of the tangent line using $y - y_1 = m(x - x_1)$. Solve for y.

As in the Section I and II, we want the x-intercept and that is when the y values is zero .... so .... we let $y = 0$ and get:

$$0 - f(x_1) = f'(x_1)(x - x_1).$$

Solve for x ↑ Remember that x_1 is **NOT** a variable. It was our first guess.

III Let's check the work for the first two sections using the calculator. We will program the calculator with the result you found above and check for accuracy.

We need to tell the computer the function we are working with so locate the Y= key and press.

Make $Y_1 =$ read: $Y_1 = (1/12)(x\text{^}3 - 7x^2 + 4x + 12)$

Make $Y_2 =$ read: $Y_2 = (1/12)(3x^2 - 14x + 4)$ (NOTE: We could have made the calculator compute the derivative for us.... in that case we would make $Y_2 =$ read: $Y_2 = nDer(Y_1, X, X)$. nDer(is found under the Math menu. Y_1 is found in the catalog or in Vars or in Y-Vars. The first X tells the calculator that the variable is x. and the second X tells the calculator to check the variable X for the value to evaluate the function at each time it computes it (rather than to only compute the value one time---- at a specific point).

On the next page, we will begin the actual program. Each line you are supposed to enter into the program will have enter this line → immediately in front of the line you are supposed to enter. The other text will be explaining what is being done and/or where to find the commands.

Turn the calculator on and press the "program" key. Use the arrow keys to move to NEW. Press Enter.
The calculator needs to know what to call the program.

enter this line → NEWTON

Press Enter.
We want the "program" to ask us for the 1st x-value to start so under the I/O menu select

enter this line → INPUT X

Note that X is the variable X notated as x-var on the calculator.
We will want the program to use what ever value it gets as the next guess so we will want the program to return to this same spot to compute the value again so we tell it this is the spot by putting a Label here:

enter this line → Lbl 1

Now we key in the formula you found on page 2.

enter this line → $X - (Y_1(X)) / Y_2(X) \rightarrow X$

This says, from the value we put in as our guess subtract the fraction computed as $f(x)$ divided by $f'(x)$ and put that new value back in the variable X.
We want to see the value the calculator found so we Display it....

enter this line → Disp X

The program will happen very quickly so we could never see the results unless we tell the machine to wait until we have see the value and specifically told it to carry on.

enter this line → Pause

After we have seen the value we want that value to be used on the next computation so we want the program to return to the spot we earlier labeled as number 1

enter this line → Goto 1

Press 2nd Quit to exit the program mode.

Every time we press Enter we will get a new x value. It is surprising how quickly that value approaches zero.

To stop the program from running press the ON button and select Quit from the two choices.

Let's test your program:

You already have the function and its derivative in the proper position so press the Program key and select NEWTON and press Enter.
You will see a question mark. The program wants to know what number you want to start with. Press 1 and press Enter.

Compare your answers from section I with the answers you are getting now. They should be the same.

Exit the program. Press Program, select NEWTON and press Enter to restart the program. This time start with $x = 5$. Compare your values. How many steps does it take to get within 8 or 10 digits of accuracy on zero?

Restart the program again. Start with –5. See how many steps it takes to get the negative one root.

IV It is time to use what we have learned.

Problem Find $\sqrt[3]{3.842}$ accurate to 3 decimal places using "Newton's Method..."

Let $x = \sqrt[3]{3.842}$

Then it follows that ..... $x^3 = 3.842$ and $x^3 - 3.842 = 0$ which could be written ...

$f(x) = x^3 - 3.842$ and now we can use or program to compute the value.
Make a list of your guess and the results and the results and the results .....

Newton's Method of finding roots has been used for several purposes over the years.

One example is when you use a function which has *NO* real roots. For example $f(x) = \frac{1}{4}x^2 + 1$. This is a parabola with only a *y*-intercept of (0,1). You could "guess" $x = 4$ and get 1.5. Next you would get – 0.583 then 3.137, then 0.931 and so on. These values "seem" to be random and were used as a method to generate ramdon mumbers.

Another use of Newton's Method is to approximate an irrational number.

If you wanted to find an approximate value for $\sqrt{3}$ you would Let $x = \sqrt{3}$. so $x^2 = 3$

It follows that $x^2 - 3 = 0$. Now we want a zero of our equation or we could phrase it as $f(x) = x^2 - 3$.

This way we want a zero for our equation. (since $f(\sqrt{3}) = 0$).

Now you want to try a guess like $x = 2$. Find $(2, f(2))$ and $f'(2)$ etc.

Now that you are an expert with Newton's Method for finding roots, do problems 1 - 13 on page 173.

Value 3 homework assignments. (This completed worksheet is worth 2 homework assignments) and all of this is due the first day after spring break (or postmarked on that date).

C 4m H Analysis

Heston's Derivative Marathon

Mr. Mumaugh

I Find the derivative of each of the following problems in set A. Locate that answer in set B . The letter with the answer goes in the blank which is marked with the problem number.

Set A

1) $(x^2-3x+8)^2$

2) $(8x-7)^{-5}$

3) $(3-2x)^{-1}$

4) $\dfrac{x}{(x^2-1)^4}$

5) $(3-2x)^2(2x-1)$

6) $\dfrac{x^2+2x}{x^2}$

7) $\left(\dfrac{3x+4}{6x-7}\right)^3$

8) $\sqrt[3]{8x^3+27}$

9) $\dfrac{5}{\sqrt[3]{x^5-32}}$

10) $\dfrac{1}{\sqrt{3x-4}}$

11) $\dfrac{x^2-4x+3}{x^{\frac{1}{2}}}$

12) $\dfrac{2x+3}{\sqrt{4x^2+9}}$

13) $\sqrt{4x^2+2x+3}$

14) $\sin(x^2+2)$

15) $\cos(4-3x)$

16) $\cos^5 3x$

17) $\sin^4 x^3$

18) $\sec(2x+1)^2$

19) $\tan(2x^2+3)$

20) $\cos(3x^2)+\cos^2 3x$

21) $\tan^3\dfrac{x}{6}$

22) $\csc^2 2x$

23) $x^2\csc^2 x^2$

24) $\tan^2 x\sec^3 x$

25) $x^2\sec^3 4x$

26) $(\sin 5x-\cos 5x)^5$

27) $\sin 4x\csc 4x$

28) $\dfrac{\cos 4x}{1-\sin 4x}$

29) $\dfrac{\sec 2x}{1+\tan 2x}$

30) $\sin\sqrt{x}+\sqrt{\sin x}$

31) $\tan\sqrt[3]{5-6x}$

32) $\cos^2\sqrt{3-8x}$

33) $\sqrt{\sin 2x-\cos 2x}$

34) $\sqrt{x^2+1}\tan\sqrt{x^2+1}$

35) $\dfrac{\cot 4x}{\sqrt{x^2+4}}$

36) $\sec\sqrt{4x+1}$

37) $\sin^2 2x\sqrt{\cos 2x}$

38) $\sin\left(\dfrac{\pi}{2}-x\right)$

39) $(2x-3)^3(3-2x)^{-1}$

40) $\sqrt{(3-x)^2-2x}$

41) $\dfrac{2(9x-2)}{135}\sqrt{(3x+1)^3}$

42) $-\dfrac{1}{3}\cos 3x+\dfrac{1}{9}\cos^3 3x$

43) $3\cos^2 4x-3\sin^2 4x$

44) $\dfrac{x}{2}-\dfrac{\sin 2x}{4}$

45) $\dfrac{1}{3}\tan 3x-x$

46) $\dfrac{1}{8}\sin 2x-\dfrac{1}{4}x\cos 2x$

47) $\dfrac{-1}{3(3x+5)}$

48) $\dfrac{1}{12}(2x^2+3)^3$

49) $\left(\dfrac{\sin x}{\sin 2x}\right)^3$

50) $\left(\dfrac{x+1}{x-1}\right)^3$

51) $\dfrac{(x+1)^2}{x-1}$

52) $\dfrac{(5-x^2)^2}{x^2-5}$

53) $\left(\dfrac{3x+2}{x+5}\right)^3$

31	42	10	25	14	35	47	7	39
9	41	22	5	21	8	34	46	1
12	49	23	18	48	30	2	50	26
6	53	37	17	43	24	45	38	11
32	13	51	27	3	36	40	15	44
19	29	52	16	33	20	28	4	/___

Part I Set B

T) $\dfrac{-2\sec^2(5-6x)^{1/5}}{(5-6x)^{2/5}}$ B) $\dfrac{1}{2}\tan^2\dfrac{x}{6}\sec^2\dfrac{x}{6}$ A) $12x^2\sin^3 x^3\cos x^3$ I) $3\sin(4-3x)$

F) $\dfrac{-4(x^2+4)\csc^2 4x - x\cot 4x}{(x^2+4)^{3/2}}$ E) $\dfrac{18-12x}{(4x^2+9)^{3/2}}$ E) $\dfrac{3}{2}x^{1/2} - \dfrac{2}{x^{1/2}} - \dfrac{3}{2x^{3/2}}$ I) $-15\sin 3x\cos^4 3x$

H) $\sin^3 3x$ E) $\dfrac{8x^2}{(8x^3+27)^{1/3}}$ N) $-48\sin 4x\cos 4x$ S) $\dfrac{1-\cos 2x}{2}$

E) $\dfrac{-3}{2(3x-4)^{3/2}}$ T) $\dfrac{x\tan\sqrt{x^2+1}}{\sqrt{x^2+1}} + x\sec^2\sqrt{x^2+1}$ D) $\sec^3 x\tan x(2\sec^2 x + 3\tan^2 x)$

D) $2x\sec^3 4x(1+6x\tan 4x)$ W) $\dfrac{1}{2}x\sin 2x$ P) $\tan^2 3x$ T) $\dfrac{2\sec 2x\,(\tan 2x - 1)}{(1+\tan 2x)^2}$

I) $2x\cos(x^2+2)$ E) $(4x-6)(x^2-3x+8)$ R) $-\sin x$ T) $2x$ A) $4x\sec^2(2x^2+3)$

F) $\dfrac{1}{(3x+5)^2}$ N) $\dfrac{3}{8}\sec^3 x\tan x$ T) $\dfrac{4\sin 2\sqrt{3-8x}}{\sqrt{3-8x}}$ T) $\dfrac{\sin 2x + \cos 2x}{\sqrt{\sin 2x - \cos 2x}}$

E) $\dfrac{-135(3x+4)^2}{(6x-7)^4}$ C) $2x\csc^2 x^2(1-2x^2\cot x^2)$ E) $\dfrac{4x+1}{\sqrt{4x^2+2x+3}}$

R) $12-8x$ O) $(8x+4)\sec(2x+1)^2\tan(2x+1)^2$ N) $\dfrac{(x-3)(x+1)}{(x-1)^2}$

E) $\dfrac{-5x^4}{(x^5-32)^{6/5}}$ N) $x(2x^2+3)^2$ D) 0 E) $\dfrac{-7x^2-1}{(x^2-1)^5}$ U) $-6x\sin 3x^2 - 3\sin 6x$

N) $x\sqrt{3x+1}$ T) $\dfrac{\sqrt{\sin x}\cos\sqrt{x} + \sqrt{x}\cos x}{2\sqrt{x\sin x}}$ E) $\dfrac{-40}{(8x-7)^6}$ E) $\dfrac{2}{(3-2x)^2}$

C) $-4\csc^2 2x\cot 2x$ N) $\dfrac{-6(x^2+2x+1)}{(x-1)^4}$ D) $25(\sin 5x - \cos 5x)^4(\sin 5x + \cos 5x)$

E) $2(3-2x)(5-6x)$ E) $\dfrac{-2}{x^2}$ R) $\dfrac{39(3x-2)^2}{(x+5)^4}$

S) $\dfrac{\sin 2x(4\cos^2 2x - \sin^2 2x)}{\sqrt{\cos 2x}}$ D) $\dfrac{4}{1-\sin 4x}$

R) $\dfrac{2\sec\sqrt{4x+1}\tan\sqrt{4x+1}}{\sqrt{4x+1}}$ S) $\dfrac{x-4}{\sqrt{x^2-8x+9}}$

II Find the equation of the tangent line to the relation given at the given point:

a) $xy+16=0$ $(-2,8)$ b) $y^2-4x^2=5$ $(-1,3)$ c) $2x^3-x^2y+y^3-1=0$ $(1,0)$

d) $3y^4+4x-x^2\sin y-4=0$ $(1,0)$ e) $xy^2+3y=27$ $(2,3)$ f) $y=2x-\dfrac{4}{\sqrt{x}}$ $(4,6)$

III Given $x \in [0,2\pi]$. Find the x-coordinates of all points on the graph of the equation $y=3x-\cos 2x$ at which the tangent line is perpendicular to the line $2x+4y=5$.

IV If $f(x)=\sin 2x-\cos 2x$ for $0\le x\le 2\pi$, find the x-coordinates of all points on the graph of f at which the tangent line is horizontal.

Industrial Strength Derivatives

Mr. Mumaugh

I A **For each of the following, find the first derivative two times. First use the product rule. Next, clear parentheses in the original expression and simplify. Then, take the derivative in the usual manner. Reconcile any differences in final answers.**

1) $8x(3x+2)$ 2) $2x^3\left(x-x^3\right)$ 3) $\left(3x^2+2x\right)(5x-2)$ 4) $3x^3(4x+8)$

5) $3x^2\left(x-\sqrt{x}\right)$ 6) $x^2\left(x^2+x^2\right)$ 7) $(3x+3)(3x-3)$ 8) $\left(2-3x^2\right)\left(x^2+2\sqrt{x}\right)$

9) $(2x+1)(1-2x)$ 10) $(2x+3)^3$ 11) $(2x+3)^4$ 12) $(2-3x)^3$

B **Use the quotient rule to find the derivative:**

1) $\dfrac{(2x-1)}{3x-2}$ 2) $\dfrac{(2x-1)^2}{3x^2-2}$ 3) $\dfrac{(2x-1)}{3x^2}$ 4) $\dfrac{2x-3}{x}$

5) $\dfrac{ax+d}{c-dx}$ 6) $\dfrac{3x-5}{5-3x}$ 7) $\dfrac{3x+2}{8-12x}$ 8) $\dfrac{5x^2+2}{2x^5-5}$

C **Find the derivative:**

1) $\dfrac{3x(2x-1)}{5x+2}$ 2) $\dfrac{(3x+2)(2x-3)}{(5x+1)}$ 3) $\dfrac{(3x+2)(2x-3)}{(3-2x)(5x+1)}$ 4) $\dfrac{(x+2)(3x-4)}{(x-2)(5x+1)}$

II A **Find the derivative twice. First clear parentheses and use the "quick derivative." Second, use the chain rule. Clear parentheses to verify that you got the same derivative.:**

1) $(3x-2)^2$ 2) $\left(3\sqrt{x}-2\right)^2$ 3) $\left(2x^2+4x+1\right)^2$ 4) $(2x+1)^4$

B **Use the chain rule to find the derivative. Note that some problems may require a combination of the product rule, quotient rule as well as the chain rule. Be sure that your work makes it perfectly clear what method you used! Show the "u"s and the "v"s.**

1) $(3x-1)^2(5-2x)^3$ 2) $\dfrac{\left(1-3x^2\right)^3}{(4x)^2}$ 3) $\dfrac{\left(x^2+1\right)^3}{3x-1}$ 4) $\dfrac{1}{(2x+3)^2}$

5) $\dfrac{1}{(2x+3)}$ 6) $x\left(\dfrac{x^2}{x+1}\right)$ 7) $\left(\dfrac{1-2x}{2+x}\right)^2$ 8) $\dfrac{(1-2x)^2}{(2+x)^2}$

9) $\sqrt{4x^2+2x+1}$ 10) $\sqrt{5-3x^2}$ 11) $\dfrac{1}{\sqrt{5-3x^2}}$ 12) $\dfrac{3x}{\sqrt{1-x^2}}$

13) $\dfrac{(x+1)^2(x-1)^2}{\left(x^2-1\right)^2}$ 14) $x(x-1)\left(x^2-x+1\right)$ 15) $\dfrac{x^2+1}{x^2}$ 16) $1+\dfrac{1}{x^2}$

17) $\sqrt{\dfrac{2x-5}{3x-8}}$ 18) $(2+3)^{17}$ 19) $\dfrac{x^2-1}{x-1}$ 20) $\dfrac{x^3-1}{x-1}$

21) $\left((2x-1)^2+2x\right)^2$ 22) $\left(4x^2-2x+1\right)^2$ 23) $16x^4-16x^3+9x^2-8x+1$

more Industrial Strength Derivatives

III Find $\frac{dy}{dx}$

a) $x^2 + y^2 = 16$

b) $x^2 + y^2 - 5x - 8y - 14 = 0$

c) $xy^2 = x + y$

d) $x^{2/3} + y^{2/3} = 8^{2/3}$

e) $x^2 - y^2 = 1$

f) $x^2 - 2xy + 4y^2 - 4x = 0$

g) $y^2 = \frac{x^2}{x^2 + 4}$

h) $5x^2 + 4xy + 3y^2 + y = 0$

i) $(x+y)^2 + (x-y)^2 = 16 + 2y$

j) $4xy + x^2 + 2y^2 = 25$

k) $x^2 = \frac{x - y^2}{x + y}$

l) $\sqrt{x+y} + \sqrt{x+2y} = 10$

IV There are six standard trig functions. What are their derivatives? (hint: $\sec x = \frac{1}{\cos x} = (\cos x)^{-1}$)

V Find $f'(x)$ two times. First take the derivative of the given as it stands. Second, change the given using a trig identity and then take the derivative. Evaluate both of your answers at $x = \frac{\pi}{4}$.

a) $f(x) = 2 \sin 2x \cos 2x$

b) $f(x) = \frac{1}{2} \sin 4x$

c) $f(x) = \frac{\tan^4 x}{\sec^2 x}$

d) $f(x) = \sin^2 2x + \cos^2 2x$

e) $f(x) = \frac{\sin 2x}{\cos x}$

f) $f(x) = (\sin^2 x + \cos^2 x)$

g) $f(x) = 1 - \sin^2 x$

h) $f(x) = \sin 2x$

i) $f(x) = 2 \sin x \cos x$

j) $f(x) = \frac{1}{2}(1 + \cos 2x)$

k) $f(x) = \frac{1}{2}(1 - \cos 2x)$

l) $f(x) = (\sin x + \cos x)^2$

m) $f(x) = \frac{\cos^2 x}{1 - \sin x}$

n) $f(x) = \sin x \tan x + \cos x$

o) $f(x) = \frac{1 + 2 \sin x \cos x}{\sin x + \cos x}$

p) $f(x) = \frac{1 - \cos 2x}{\sin 2x}$

q) $f(x) = \sec^2 x$

r) $f(x) = \frac{2 \tan x}{1 + \tan^2 x}$

VI Find $f'(x)$. Do not simplify. Be sure to do all the pieces in the proper order.

a) $f(x) = \left(\frac{\sqrt{3x^2 - x}}{5x^3 + 1} \right)^{\frac{2}{3}}$

b) $f(x) = \frac{(3x^3 + 2)(2x - 3)^2}{\sqrt{5x^3 + 1}}$

c) $f(x) = \frac{5(\sin^2 x - \cos x)^3}{6\sqrt{\sin x \cos x}}$

VII Prepare proper numberlines for each of the following. Sketch a graph of f. Identify extrema and PI. Justify as usual. The stated intervals are "windows of view" and not necessarily the valid domains.of the functions.

a) $f(x) = \sin x + x \; ; \; -2\pi \le x \le 2\pi$

b) $f(x) = \sin x + \cos x \; ; \; -2\pi \le x \le 2\pi$

c) $f(x) = \frac{\cos x}{1 + \sin x} \; ; \; -2\pi \le x \le 2\pi$

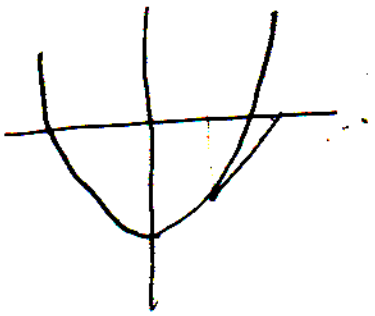
d) $f(x) = \sin\left(\frac{x^2}{1 + x^2} \right) \; ; \; -2\pi \le x \le 2\pi$

e) $f(x) = \sqrt{x - x^2} \; ; \; -2 \le x \le 2$

f) $f(x) = x^6 - x^4 \; ; \; -2 \le x \le 2$

Pg 173

method (1) $\sqrt{2}$



$x = \sqrt{2}$

$x^2 - 2 = f(x)$

Guess #1

$x = 1 \quad f' = 2x\Big|_{x=1} = 2$

$(1, -1) \quad m = 2$

$y + 1 = 2(x - 1)$

$y = 2x - 3 \overset{\text{Set } 0}{=}$

$x = \frac{3}{2}$

$f' = 2x\Big|_{\frac{3}{2}} = 3$

$\frac{9}{4} - \frac{6}{4} = \frac{1}{4}$

$m = 3 \quad \left(\frac{3}{2}, \frac{1}{4}\right)$

$y - \frac{1}{4} = 3\left(x - \frac{3}{2}\right)$ $\frac{18}{4}$

$y - \frac{1}{4} = 3x - \frac{9}{2}$

$y = 3x - \frac{17}{4}$

$x = \frac{17}{12}$

method (2)

$$x_{N+1} = \frac{1}{2}\left(x_N + \frac{a}{x_N}\right)$$

$x_2 = \frac{3}{2} = \frac{1}{2}\left(1 + \frac{2}{1}\right)$

$x_3 = \frac{1}{2}\left(\frac{3}{2} + \frac{2}{\frac{3}{2}}\right)$

$x_3 = \frac{1}{2}\left(\frac{3}{2} + \frac{4}{3}\right)$

$= \frac{9+8}{12}$

$x_3 = \frac{17}{12}$

H. Analysis

unCHAINed Melody

Mr. Mumaugh

(As Al Hibbler intended it to be...)

Find the derivatives and evaluate at the given values using the following table.
Transfer the letters to the spaces provided for a final clue for solving this puzzle.

Example 1:

$$\left(h(g(x))\right)'\Big|_{x=2} = h'(g(2)) \cdot g'(2)$$

$$h'(4) \cdot 3$$

$$3 \cdot 3 = 9$$

Example 2:

$$\left(f(g(h(x)))\right)'\Big|_{x=2} = f'(g(h(2))) \cdot g'(h(2)) \cdot h'(2)$$

$$f'(g(4)) \cdot g'(4) \cdot 2$$

$$f'(3) \cdot 2 \cdot 2$$

$$2 \cdot 2 \cdot 2 = 8$$

$x \downarrow$	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$	$h(x)$	$h'(x)$
2	3	4	4	3	4	2
3	4	2	2	4	3	4
4	4	3	3	2	2	3

R) $\left(g(h(x))\right)'\Big|_{x=2}$

N) $\left(g(f(x))\right)'\Big|_{x=4}$

S) $\left(f(h(x))\right)'\Big|_{x=3}$

F) $\left(f(g(x))\right)'\Big|_{x=2}$

H) $\left(g(g(x))\right)'\Big|_{x=3}$

V) $\left(h(f(x))\right)'\Big|_{x=2}$

T) $\left(g(h(f(x)))\right)'\Big|_{x=3}$

D) $\left(h(g(f(x)))\right)'\Big|_{x=4}$

O) $\left(h(f(g(x)))\right)'\Big|_{x=2}$

A) $\left(g(f(h(x)))\right)'\Big|_{x=4}$

E) $\left(f(g(f(x)))\right)'\Big|_{x=2}$

I) $\left(f(g(f(g(x))))\right)'\Big|_{x=3}$

We will never forget the ...

24	64	4	256	16	48	18
256	16	64	27	9	18	12
64	256	6	8	256	24	64